

Problem Solving Fall 2010  
Central Connecticut State University Department of Mathematical  
Sciences

**I. Course Title:** Problem Solving I, II, III  
Math 120, 220, 320  
1 credit

**II. Course Details:**

Section: Math 120,220,320 meets Wednesdays 9-10:40 AM in Frank J. Diloreto 002  
Instructor: J. Bradlau  
e-mail: jbradlau@snet.net  
Phone 860-628-9327  
Office: Frank J. Diloreto

Office Hours: Monday, Wednesday, Friday 7:30-8 AM Frank J. Diloreto 107  
Monday 9-10 AM TBA  
Friday 9-9:30 AM TBA

**III. Students for Whom the Course is Intended:**

Secondary Mathematics (B.S.) majors. This is one of three one-credit seminars (MATH 120, MATH 220, and MATH 320), which are required for the B.S. degree for students matriculating in Fall 1998 and later. It is also recommended for B.S. (Secondary) majors who matriculated prior to Fall 1998 and may be taken as an elective by B.S. (Elementary) majors.

**IV. Attendance:**

Since this is a seminar, which meets only once a week and interaction among students is an essential part of the course, attendance at every class is expected. Absences will adversely affect the class participation portion of the final grade.

**V. Prerequisites:**

For Math 120: Math 115 (C- or higher) or Math 119 (C- or higher) or Placement Exam.

For Math 220: Math 120 (C- or higher) and Math 122 (C- or higher).

For Math 320: Math 220 (C- or higher) and Math 228 (C- or higher).

**VI. Textbook:**

There is no textbook for this course. You will be provided with handouts containing problems to be worked on. In addition, a list of resources for additional problems is attached to the syllabus.

**VII. Work load:**

The norm for university courses is that a minimum of two hours of homework per week is required for every credit hour. Consequently, for this class you are expected to spend at least 2 hours per week outside of class.

**VIII. Expectations:**

All three seminars, MATH 120, MATH 220, and MATH 320 meet at the same time. Students in

MATH 220 and MATH 320 will have had the experience of taking this seminar at least once previously. In addition, in most cases they will have completed more of the standard courses in the major. For instance, all students in MATH 220 will have at least one semester in calculus. Students in MATH 320 will have completed courses in discrete mathematics and linear algebra. The mixing of the three groups is deliberately designed to enable students in MATH 120 to learn from more experienced problem solvers.

Because of the heterogeneous nature of the class, expectations are set on an individual basis. You will be given a wide variety of problems to choose from. Work on those that you find challenging, but not impossible. The problems will be weighted according to the level of difficulty. The easiest problems will be worth 1 point each, more difficult problems 2 points each and so on.

### **IX. Portfolios:**

On four occasions during the semester, you will submit a portfolio containing detailed solutions to some problems you have worked over the past few weeks. The problems submitted must carry a total weight of at least 6 points for Math 120 students, 7 points for Math 220, and 8 points for Math 320. Along with each solution you will reflect on the process you went through to solve the problem, using Polya's four steps as a framework. We are particularly interested in problems that prove challenging to you, those for which your initial approach was unfruitful, and situations that lead to significant insights and discoveries. The problems submitted in the portfolio may include some you have worked on by yourself and some you have discussed with classmates. Please acknowledge instances where other class members or the instructor have contributed to the solution or where you have used hints provided by the author of the textbook or other sources. Portfolios will be assessed for the level of difficulty of the problems, the accuracy and elegance of the solutions, and your analysis of the problem solving process. All portfolio solutions must be typed. You must use the Equation Editor and Geometer's Sketchpad for a professional looking document. A sample-grading sheet is attached to this syllabus.

### **X. Final Presentation:**

In place of a final examination, you will choose one problem to present to the class on Dec. 1, or Dec. 8, or Dec. 13. You will give a ten-minute oral presentation explaining your solution, including Polya's four-step problem solving process. A sample grading sheet is attached to this syllabus.

### **XI. Class participation:**

Your class participation will be graded. An important part of this grade will be oral presentations of problems that carry at least five points and **are not included** in your portfolios. You are advised to make these presentations early and throughout the course.

### **XII. Evaluation:**

Your grade for the course will be determined on the following basis:

Class participation	20%
Portfolios (four, 15% each)	60%
Final presentation	20%
Total:	100%

## Calendar

Wednesday, September 1, 2010	class, introductory thoughts about problem solving
Wednesday, September 8, 2010	class, introduction to Equation Editor and Geometer's Sketchpad for portfolio preparation
Wednesday, September 15, 2010	class
Wednesday, September 22, 2010	Portfolio #1 Due
Wednesday, September 29, 2010	class
Wednesday, October 6, 2010	class
Wednesday, October 13, 2010	Portfolio #2 Due
Wednesday, October 20, 2010	class
Wednesday, October 27, 2010	class
Wednesday, November 3, 2010	Portfolio #3 Due
Wednesday, November 10, 2010	class
Wednesday, November 17, 2010	class
Wednesday, December 1, 2010	Final Presentations begin
Wednesday, December 8, 2010	Final Presentations, Portfolio #4 due
Monday, December 13, 2010	Final Presentations (if needed) 8-10 AM

## University Policies:

1. If you need course adaptations or accommodations because of a disability, if you have emergency medical information to share with me, or if you need special arrangements in case the building must be evacuated, please make an appointment with me as soon as possible. My telephone numbers and office hours are given above.

2. In the event of a weather emergency, which requires curtailment or cancellation of classes, listen to WTIC (1080 AM) or call 832-3333 for the "general snow message." In the event that I am prevented from traveling to campus and must cancel class, I will send the class an e-mail message. Last day to drop the course is March 19. Withdrawal forms are available in the Enrollment Center, Willard Hall. Cessation of attendance, notice to the instructor, or telephone calls to the Enrollment Center are not considered official notice of a student's intention to withdraw from the course.

3. All students are expected to demonstrate integrity in the completion of their coursework. Academic integrity means doing one's own work and giving proper credit to the work and ideas of others. It is the responsibility of each student to become familiar with what constitutes academic dishonesty and plagiarism and to avoid all forms of cheating and plagiarism. Students who engage in plagiarism and other forms of academic misconduct will face academic and possibly disciplinary consequences. Academic sanctions can range from a reduced grade for the assignment to a failing grade for the course. From a disciplinary standpoint, an Academic Misconduct Report may be filed and a Faculty Hearing Board may impose sanctions such as probation, suspension or expulsion.

For further information on academic misconduct and its consequences, please consult the Student Code of Conduct (<http://www.ccsu.edu/StudentConduct>) and the Academic Misconduct Policy (<http://www.ccsu.edu/AcademicIntegrity>).

## Resources:

### 1. Works about Problem Solving and the Teaching or Problem Solving

Brown, Stephen I. and Walter, Marion I. *The Art of Problem Posing*, 2nd edition. Hillsdale, NJ: Lawrence Erlbaum, 1990.

Charles, Randall I. and Silver, Edward A. *The Teaching and Assessing of Mathematical Problem Solving*. Reston, VA: National Council of Teachers of Mathematics, 1989.

Krantz, Steven V. *Techniques of Problem Solving*. Providence, RI: American Mathematical Society, 1997.

Krulik, Stephen and Rudnick, Jesse A. *Problem Solving: A Handbook for Secondary Teachers*. Boston, MA: Allyn & Bacon, 1989.

Krolik, Stephen. *Problem Solving in School Mathematics: 1980 Yearbook*. Reston, VA: National Council of Teachers of Mathematics, 1980.

Polya, George. *How to Solve It*

Schoenfeld, Alan H. *Problem Solving in the Mathematics Curriculum: A Report. Recommendations. and an Annotated Bibliography*. Washington, DC: Mathematical Association of America, 1983.

### 2. Sources of Problems

#### a. Periodicals

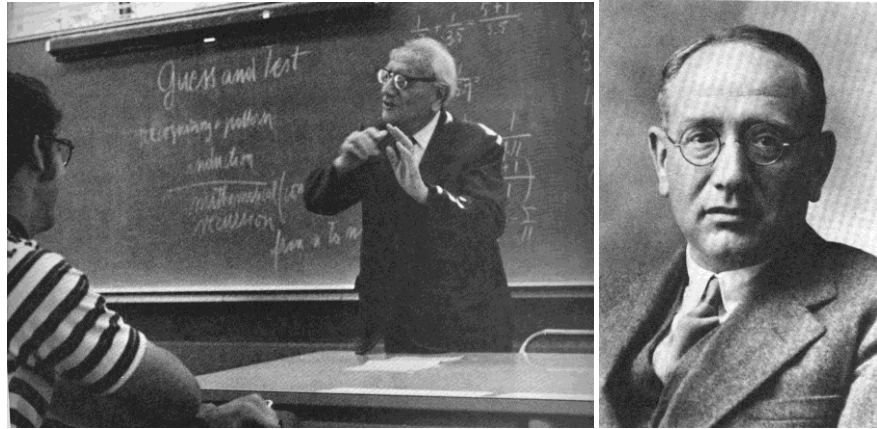
College Mathematics Journal  
Mathematics Magazine  
Mathematics Teacher  
Quantum

#### b. "Stella Project," a collection of over 1200 non-routine problems. Unpublished database courtesy of Rudd Crawford, Oberlin College, Oberlin, OH.

#### c. Web Sites

National Council of Teachers of Mathematics, <http://www.nctm.org>  
Eisenhower National Clearinghouse for Mathematics and Science Education,  
<http://www.enc.org/>  
Shack's Page of Math Problems, <http://www.charm.net/~shack/math/>  
Math Forum Internet Collection of Math Puzzles, <http://forum.swarthmore.edu/~stevesteve/mathpuzzles.html>  
Eric's Treasure Trove of Mathematics,  
<http://www.astro.virginia.edu/~eww6n/math/math.html>

**How to Solve It**  
**G. Polya (1887-1985)**  
**Professor of Mathematics at Stanford University**



The Art of Problem Solving – from an interview with George Polya in 1977

**MP:** That brings up my next question. How did you become interested in heuristics and the art of problem solving? Did anyone or any event influence you on this?

**Polya:** Well, I think I wrote it somewhere. In one of my books it is mentioned. I came very late to mathematics. I had an interest in biology, literature and philosophy. And as I came to mathematics and learned something of it, I thought: Well, it is so, I see, the proof seems to be conclusive, but how can people find such results? My difficulty in understanding mathematics: How was it discovered? And then I was deeply influenced by some books. I wish to mention just two. One was the book of Ernst Mach on the history of mechanics. For me personally this was the most beautiful book I read. I read it at the right time because I knew a little physics, but just a little. I was just right for it. His main theme is: You cannot understand a theory unless you know how it was discovered. His best book and best-known book is on mechanics, but he wrote also other books, on the theory of heat and still others. But that was the main idea: In order to understand a theory really, you must know how it was discovered. So he came to heuristics. In fact, in some of his other books there are a few direct remarks on problem solving. Then I thought about it and I came across the *Regulae* of Descartes, which is really a book on problem solving. That is not mentioned in any history of philosophy, because those historians who wrote about him didn't know about problem solving. My interest in literature contributed a little. When I was interested in literature, I was most interested in books of Hippolyte Taine and he wrote about literature in a quasi-scientific way. How in such a vague subject you can bring in something that approximates science, that deeply impressed me. It also contributed to my interest in heuristics. It is essentially a vague question, and that you can introduce something which has something to do with science, that I think I learned from Taine. I was also impressed by his style.

George Polya was interviewed by G.L. Alexanderson at Polya's 90th birthday party at Stanford, May 13, 1977. You can read more about this interview in *Mathematical People: Profiles and Interviews*. Burkhauser Boston, Inc., 1985. The pictures are from this source too.

**Polya's problem solving steps:**

**First. You have to understand the problem.**

UNDERSTANDING THE PROBLEM

What is the unknown? What are the data? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory? Draw a figure. Introduce suitable notation. Separate the various parts of the condition. Can you write them down?

**Second. Find the connection between the data and the unknown.**

DEVisING A PLAN

Have you seen it before? Or have you seen the same problem in a slightly different form? Do you know a related problem? Do you know a theorem that could be useful? Look at the unknown! And try to think of a familiar problem having the same or a similar unknown. Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible? Could you restate the problem? Could you restate it still differently? Go back to definitions.

If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other? Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

**Third. Carry out your plan.**

CARRYING OUT THE PLAN

Carrying out your plan of the solution, check each step. Can you see clearly that the step is correct? Can you prove that it is correct?

**Fourth. Examine the solution obtained.**

LOOKING BACK.

Can you check the result? Can you check the argument? Can you derive the result differently? Can you see it at a glance? Can you use the result, or the method, for some other problem? DO NOT JUST LOOK OVER YOUR WORK AND SAY IT LOOKS CORRECT!!

Problem solving heuristics from George Polya, Princeton University Press, 1973

Mathematics 120/220/320 Portfolio Assessment

Name \_\_\_\_\_

Assessment	Possible Points	Student's points
1. The portfolio is complete in that it contains the solutions of problems with a total number of points appropriate to the level of the course (6 for 120, 7 for 220, 8 for 320).	10	_____
2. Solutions to the problems are accurate and complete.	20	_____
3. The student uses Polya's four steps effectively to solve problems.	15	_____
4. The student demonstrates significant insight and creativity.	10	_____
5. The selection includes items which were challenging to the student.	10	_____
6. The writing is clear and easy to understand.	10	_____
7. The writing is free of errors in the use of mathematical terminology.	15	_____
8. The writing is free of grammatical and spelling errors.	10	_____
Total	100	grade _____

## Problem Solving Evaluation of Presentation

Presenter:

Score: \_\_\_\_\_ of 100

1. 60 points Application of Polya's problem solving strategies
  - a. \_\_\_\_\_ Understanding the problem (15 points)
    - i. \_\_\_\_\_ What is the unknown?
    - ii. \_\_\_\_\_ What are the conditions?
    - iii. \_\_\_\_\_ Are the conditions sufficient?
    - iv. \_\_\_\_\_ Draw a figure
    - v. \_\_\_\_\_ Introduce notation
  - b. \_\_\_\_\_ Devise a plan (15 points)
    - i. \_\_\_\_\_ Have you seen it before? Or in a slightly different form?
    - ii. \_\_\_\_\_ Do you know a useful theorem?
    - iii. \_\_\_\_\_ Do you know a related problem?
    - iv. \_\_\_\_\_ Did you include all the data? All the conditions?
  - c. \_\_\_\_\_ Carry out the plan (15 points)
    - i. \_\_\_\_\_ Followed the plan
    - ii. \_\_\_\_\_ Check that each step is done correctly?
    - iii. \_\_\_\_\_ Can you prove that your solution is correct?
  - d. \_\_\_\_\_ Looking back (15 points)
    - i. \_\_\_\_\_ Can you check your result?
    - ii. \_\_\_\_\_ Can you check the argument?
    - iii. \_\_\_\_\_ Is there another way to see the result at once? (elegance)
    - iv. \_\_\_\_\_ Can you use the result or the method for some other problem?
  
2. 40 points Presentation methods
  - a. \_\_\_\_\_ Visual resources (looking for effectiveness of use, not a particular resource.) Some options are listed below:
    - i. Chalk board
    - ii. Overheads
    - iii. Handouts
    - iv. Models
    - v. Other
  - b. \_\_\_\_\_ Timing planned effectively
  - c. \_\_\_\_\_ Voice pleasant, easy to hear
  - d. \_\_\_\_\_ Used correct Mathematical terminology and English grammar
  - e. \_\_\_\_\_ Aware of the audience
  - f. \_\_\_\_\_ Presentation planned to respond to the interest and ability level of the audience
  
3. General Comments