MATH 383 – College Geometry – Practice Test 1

Spring 2010

This test is worth 100 points. The take-home part of the test is worth 40 points. You have to solve a total of six problems:
One problem from the set 1 - 4.
One problem from the set 5 - 7.
Four problems from the set 8 - 20.

Each of the problems you select to solve is worth 10 points.

Problem 1. *Use the first five propositions of Euclid's Book I of the Elements to prove that the diagonals of a rhombus bisect each other and are perpendicular.*

Problem 2. *Let A, B, C, D be four distinct points such that no three of them are collinear. Assume further that $AB = BC$ and that $CD = DA$. Use the first five propositions of Book I to prove that if the segments $\overline{AC}$ and $\overline{BD}$ intersect then the lines $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are perpendicular.*

Problem 3. *Let A and B be any two distinct points. Explain how to find a point C such that $AB = \frac{1}{3}AC$ using only straightedge and compass.*

Problem 4. *Explain how to construct a right triangle $\triangle ABC$ such that $BC = \frac{4}{3}AB$ using only compass and straightedge.*

Problem 5. *Prove that an incidence geometry in which all lines contain at least three points must have at least 7 points.*

Problem 6. *Interpret* point *to mean one of the five vertices of a pentagon,* line *to mean one of the sides of the pentagon, and* lie on *to mean that the vertex is an endpoint of the side. Which incidence axioms hold in this interpretation? Which parallel postulates hold in this interpretation?*

Problem 7. *Consider an incidence geometry that satisfies the Euclidean parallel postulate and has a finite number of points. Let ℓ, m be two distinct lines and P a point that is not contained in ℓ and is not contained in m. Is it true that the number of points of ℓ is the same as the number of points of m? If your answer is yes, provide a proof. If your answer is no, provide a counterexample.*

Problem 8. *The square metric is the function $D : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by*

$$D((x_1, y_1), (x_2, y_2)) = \max\{|x_2 - x_1|, |y_2 - y_1|\}.$$

(a) Prove that D is a metric according to the definition in Venema's book. (b) Let ℓ be the line $y = 3x + 7$. Find a coordinate function for ℓ relative to the metric D.

Problem 9. *Prove the Ruler Placement Postulate (Theorem 5.4.14).*

Problem 10. *Explain why every model of neutral geometry must contain infinitely many points and infinitely many lines.*

Problem 11. *Let* A, B *be two distinct points. Prove that* $\overrightarrow{AB} \cup \overrightarrow{BA} = \overleftrightarrow{AB}$.

Problem 12. *Let* A, B *be two distinct points. Prove that* $\overline{AB}$ *is a convex set.*

Problem 13. *Let* A, B, C, D *be four distinct points of the plane such that* $A \star B \star C$ *and* $B \star C \star D$. *Use the Betweenness Theorem for Points to prove that* B *and* C *lie between* A *and* D.

Problem 14. *Let* A, B, C, D, E, F *be six points such that* $A \star B \star C$, $D \star E \star F$, $\overline{AB} \cong \overline{DE}$, *and* $\overline{BC} \cong \overline{EF}$. *Prove that* $\overline{AC} \cong \overline{DF}$.

Problem 15. *Consider three noncollinear points* A, B, C, *and let* D *be any point that is in the interior of the two angles* $\angle ABC$ *and* $\angle BCA$. *Prove that* D *is also in the interior of the angle* $\angle CAB$.

Problem 16. *Betweenness on the sphere is defined in Problem 5.25. Let* A, B, C, D *be any four distinct points of the sphere such that* $A \star B \star C$ *and* $B \star C \star D$. *Is it true that* B *and* C *must be between* A *and* D? *If your answer is yes, prove it. If your answer is no, provide a counterexample.*

Problem 17. *Let* A, B, C, D *be four distinct points such that* A, B, C *are noncollinear and such that the midpoint of the segment* $\overline{AB}$ *is the same as the midpoint of the segment* $\overline{CD}$. *Prove that* B, C, D *are noncollinear.*

Problem 18. *Let* A, B, C, D *be four distinct points such that* A, B, C *are noncollinear and such that the midpoint of the segment* $\overline{AB}$ *is the same as the midpoint of the segment* $\overline{CD}$. *Prove that* $\overline{BC} \cong \overline{AD}$.

Problem 19. *Let* A, B, C, D *be four distinct points such that* A, B, C *are noncollinear and such that the midpoint of the segment* $\overline{AB}$ *is the same as the midpoint of the segment* $\overline{CD}$. *Prove that* $\triangle ADC \cong \triangle BCD$.

Problem 20. *Explain why the taxicab metric cannot be used as metric for a model of neutral geometry. Support your answer with an specific counterexample.*