MATH 135 Lecture Notes
Related Rates

Five steps for Solving Related Rates Problems (page 132).

Step 1. **Understand** the problem by reading it carefully.

Step 2 Try to **visualize** the situation. If possible make a sketch.

Step 3 From the information given you should be able to **assign** variables to the quantities whose rates are related and determine an **equation**. The equation may or may not be directly stated in the problem.

Step 4. **Differentiate** each side of the equation with respect to time, or other desired variable. (Notice that here you are inevitably using the chain rule, though this is not mentioned explicitly in the text. See explanations for Examples 1, 3, and 4 below.)

Step 5. After differentiating, substitute all given rates and given values of variables into the equation. **Solve** for the unknown rate of change. Indicate appropriate units (as modeled especially in Examples 1 and 2) and **answer** the question. **Check** your result.

Example 1 (page 130) *E* is voltage, *T* is temperature, *t* is time.

$$E = -2.800T + 0.006T^2$$

$\frac{dE}{dt} = \frac{dE}{dT}\frac{dT}{dt}$ This is the chain rule. The text leaves out this step.

$\frac{dE}{dt} = (-2.800 + 0.012T)\frac{dT}{dt}$ The expression in parentheses is $\frac{dE}{dT}$.

Now, notice that in order to answer the question, you must substitute the information that is given, namely $T = 100^o C$ and $\frac{dT}{dt} = 1.00\frac{^oC}{\min}$ in order to get $\frac{dE}{dt} = 4.00\frac{V}{\min}$.

Example 3 (page 131) *V* is volume, *r* is radius, *t* is time.

$$V = \frac{4}{3}\pi r^3$$

$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt}$ Again, this is the chain rule.

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$ because $\frac{dV}{dr} = 4\pi r^2$.

Notice that in this case you are given r and $\frac{dV}{dt}$. We must solve for $\frac{dr}{dt}$.

Example 4 (page 131). F is force, r is distance from the center of the earth, and t is time.

Note that this is a variation problem so we must first solve for k.

The chain rule is used here in the form $\frac{dF}{dt} = \frac{dF}{dr}\frac{dr}{dt}$.

Homework, with hints:

Pages 132-133:

1 is similar to Example 1.

3. you'll have to use the chain rule to find $\frac{dD}{dx}$.

Then apply the chain rule again to find $\frac{dD}{dt} = \frac{dD}{dx}\frac{dx}{dt}$. Note that $\frac{dx}{dt}$ is the plane's speed.

9 use the formula for the area of a circle.

11. Volume of a cube with edge e is given by $V = e^3$.

13 another inverse variation example like Example 4.

15 is similar to Example 3.

23 requires a proportion using similar triangles. Let s represent the length of the shadow and x the distance from the man to the light pole. Solve for s in terms of x

In addition, here are two more max-min problems.

Pages 149 and 150: 27 and 31.