MATH 135 Lecture Notes
December 5, 2006

Sections 2.5 & 2.6
Ellipses and Hyperbolas

1. Equation of an ellipse (center at the origin, **foci** on x-axis) 2-17 on page 47:

(1) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ It is assumed that $a > b$.

From this equation we can easily see that the x-intercepts are (-a, 0) and (a, 0). (These points are also called **vertices**. The line segment joining them is called the **major axis**.

We can also see that the y-intercepts are (0, -b) and (0, b). The line segment joining them is called the **minor axis**.

The **foci** are the points (-c, 0) and (c, 0) where (2) $c^2 = a^2 - b^2$. For every point on the ellipse the sum of the distances to the two foci is 2a. (The proof is on page 47—we'll just verify that this is true for the x- and y-intercepts. Study figure 2-56 on page 48.

2. Parametric equations for an ellipse: (3) $x = a\cos t$ and $y = a\sin t$. Use these to draw an ellipse on a calculator. Check to see that these equations work by substituting into equation (1) and getting the identity $\cos^2 t + \sin^2 t = 1$.

3. Equation of a hyperbola (center at the origin, **foci** on x-axis) 2-20 on page 52:

(4) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ It is assumed that $a > b$. Compare with equation (1).

From this equation we can easily see that the x-intercepts are (-a, 0) and (a, 0). (These points are also called **vertices**. The line segment joining them is called the **transverse axis**.

This hyperbola does intersect the y-axis. However, the line segment joining (0, -b) and (0, b) is called the **conjugate axis**.

The **foci** are the points (-c, 0) and (c, 0) where $c^2 = a^2 + b^2$. For every point on the hyperbola the difference of the distances to the two foci is 2a. (Compare with the equation (2) and the focal property of the ellipse.)

Note that a hyperbola has two branches and two asymptotes. Study figure 2-69 on page 54.

4 Hyperbolic functions. (See page 271 Exercise 47)

The **hyperbolic sine** of t is defined as $\sinh t = \frac{1}{2}(e^t - e^{-t})$. Pronounced "sinch"

The **hyperbolic cosine** of t is defined as $\cosh t = \frac{1}{2}(e^t + e^{-t})$.

For all values of $t, \cosh^2 t - \sinh^2 t = 1$. (Prove using the definitions of cosh and sinh.)

5. . Parametric equations for a hyperbola: (5) $x = a\cosh t$ and $y = a\sinh t$. Use these to draw one branch of a hyperbola on a calculator. Check to see that these equations work by substituting into equation (4) and getting the identity $\cosh^2 t - \sinh^2 t = 1$. Compare the parametric equations of ellipses and hyperbolas.

6. Foci on the y-axis. For both the ellipse and the hyperbola, change the role of *x* and *y*.

Ellipse: (6) $\dfrac{y^2}{a^2} + \dfrac{x^2}{b^2} = 1$. Again $a > b$ and $c^2 = a^2 - b^2$. Foci are (0, -*c*) and (0, *c*).

See equation (2-19) and figure 2-57 on page 48.

Hyperbola: (7) $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$. Again $a > b$ and $c^2 = a^2 + b^2$ Foci are (0, -*c*) and (0, *c*).

See equation (2-24) and figure 2-70 on page 54.

Study Examples 1-4 on pages 48 and 49 and Examples 1-3 on pages 54 and 55.

Exercises:

Page 51: 1, 3, 5, 7, 13, 15, 31, 33

Page 57: 1, 3, 5, 13, 15, 31