MATH 135 Lecture Notes | Sections 2.4 & 2.7
December 7, 2006 | Parabolas and Translation of Axes

<u>Definition of parabola</u> (page 42): A **parabola** is defined as the locus of a point (x, y) that moves so it is always equidistant from a given line (called the **directrix**) and a given point (called the **focus**)

<u>Equations for Parabolas</u>:

For a parabola with focus at (p,0) and directrix $x = -p$: $y^2 = 4px$ (Equation 2-15, p. 43)

For a parabola with focus at (0, p,) and directrix $y = -p$: $x^2 = 4py$ (Equation 2-16, p. 44)

Note 1: In both cases, the origin is half way between the focus and the directrix and is the **vertex** of the parabola.

Note 2: In the study of quadratic functions we often begin with a function of the form $y = ax^2$.
This is equivalent to Equation 2-16 with $a = \dfrac{1}{4p}$.

<u>Translation of Axes</u>:

Equations (2-28) through (2-33) give equations for parabolas, ellipses, and hyperbolas for which the axes have been "translated". Compare these equations to the standard forms introduced earlier. In each case x is replaced by $x - h$ and y is replaced by $y - k$.

Also look at equation of a circle on page 38, equation 2-11.

For the parabola, the point (h, k) is the vertex (midway between focus and directrix).
For the ellipse, circle, and hyperbola, the point (h, k) is the center (midway between foci)

Circle: $(x-h)^2 + (y-k)^2 = r^2$

Parabola: $(y-k)^2 = 4p(x-h)$ $\qquad$ $(x-h)^2 = 4p(y-k)$

Ellipse: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$ $\qquad$ $\dfrac{(y-k)^2}{a^2} + \dfrac{(x-h)^2}{b^2} = 1$

Hyperbola: $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$ $\qquad$ $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

<u>Completing the square:</u> It is often possible to get a second degree equation in x and y into one of the above forms, using the technique of **completing the square**.

Example 5 page 40

$x^2 + y^2 - 6x + 8y - 24 = 0$	
$x^2 - 6x + y^2 + 8y = 24$	rearranging terms
$x^2 - 6x + 9 + y^2 + 8y + 16 = 24 + 9 + 16$	adding 9 and 16 to complete the squares
$(x^2 - 3)^2 + (y + 4)^2 = 49$	rewriting as squares of binomials

This is the equation of a circle with center (3, -4) and radius = 7.

Example 3 page 59

$2x^2 - y^2 - 4x - 4y - 4 = 0$

Since the x^2 and y^2 terms have opposite signs, we have a hyperbola.

$2x^2 - 4x - y^2 - 4y = 4$	rearranging terms
$2(x^2 - 2x) - (y^2 + 4y) = 4$	taking out common factors
$2(x^2 - 2x + 1) - (y^2 + 4y + 4) = 4 + 2 - 4$	adding 2 and -4 to complete the squares
$2(x - 1)^2 - (y + 2)^2 = 2$	rewriting as squares of binomials
$\frac{(x-1)^2}{1} - \frac{(y+2)^2}{2} = 1$	dividing to get standard form for equation

The center of the hyperbola is (1, -2).

Exercises: page 41: 21, 25
page 46: 1, 3, 5, 7, 13, 15, 17
page 60: 1, 3, 5, 7,21 23, 25