MATH 135 Lecture Notes Section 3.5 Derivatives of Polynomial Functions
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The definition of derivative is used to prove each of these "shortcut" rules for finding derivatives. We express these rules with both *dy/dx* and *f(x)* notation.

1. Derivative of a constant (3-8, page 91)

   $$\frac{dc}{dx} = 0 \qquad \text{If } f(x) = c\text{, a constant, then } f'(x) = 0.$$

2. Derivative of a positive integral power (3-9, page 91)

   $$\frac{dx^n}{dx} = nx^{n-1} \qquad \text{If } f(x) = x^n\text{, then } f'(x) = nx^{n-1}.$$

   We will derive this rule using the delta process and looking for a pattern:

   If $f(x) = x$, then $f'(x) = 1 = 1x^0$.
   If $f(x) = x^2$, then $f'(x) = 2x^1$.
   If $f(x) = x^3$, then $f'(x) = 3x^2$.
   If $f(x) = x^4$, then $f'(x) = 4x^3$.

   Pascal's triangle is used to show that the pattern holds, in general for any positive integer *n*.

3. Derivative of a constant times a function (3-10, page 92)

   $$\frac{d(cu)}{dx} = c\frac{du}{dx} \qquad \text{If } g(x) = c\,f(x)\text{, then } g'(x) = c\,f'(x).$$

4. Derivative of a sum (3-11. page 93)

   $$\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \text{If } h(x) = f(x) + g(x)\text{, then } h'(x) = f'(x) + g'(x).$$

Study example 6 on page 93 to see how these rules are applied to a polynomial function:

Start with $y = 2x^4 - 6x^2 - 8x - 9$

Apply rule 4 to get: $$\frac{dy}{dx} = \frac{d(2x^4)}{dx} - \frac{d(6x^2)}{dx} - \frac{d(8x)}{dx} - \frac{d(9)}{dx}$$

Apply rule 3 to get $$\frac{dy}{dx} = 2\frac{dx^4}{dx} - 6\frac{dx^2}{dx} - 8\frac{dx}{dx} - \frac{d(9)}{dx}$$

Now apply rule 2 to the first three terms and rule 1 to the last term:

$$\frac{dy}{dx} = 2(4x^3) - 6(2x) - 8(1) - 0 = 8x^3 - 12x - 8$$

A word of advice: When applying these rules, at first you are less likely to make errors if you apply one rule at a time. With practice you will be able to combine steps.

More on Example 6. Notice that once you have found an expression for the derivative, you may then evaluate it for a particular value of x, in this case $x = -2$.

In this case $f(x) = 2x^4 - 6x^2 - 8x - 9$, so
$f(-2) = 2(-2)^4 - 6(-2)^2 - 8(-2) - 9 = 2*16 - 6*4 + 8*2 - 9 = 32 - 24 + 16 - 9 = 15.$

Also $f'(x) = 8x^3 - 12x - 8$, so
$f'(-2) = 8(-2)^3 - 12(-2) - 8 = 8*(-8) + 24 - 8 = -64 + 24 - 8 = -48.$

Geometric interpretation: The graph of the function contains the point (-2, 15).
The slope of the tangent line at this point is -48.
For more on tangent lines, see Example 7.

Check by graphing Y1 = 2X^4 – 6x^2 – 8x – 9 with the window
Xmin = -4.7, Xmax = -4.7, Xscl = 1, Ymin = -50, Ymax = 50, Yscl = 10.
TRACE to the point (-2,15) and use 2ndCALC dy/dx to find the derivative.

Example 8, page 94 shows how the derivative is applied to a problem involving displacement to find the instantaneous velocity.

Exercises: pages 94-5: 1, 3, 5, 7, 9, 11, 15, 21, 23, 25, 27, 29, 33, 35, 41