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SUPERNEIGHBOURHOOD SPACES AND EXTENSIONS OF TOPOLOGICAL SPACES

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Иван Гочев. **Суперокрестностные пространства и расширения топологических пространств.** В этой работе вводится понятие суперокрестностного пространства и при помощи этого понятия описываются два класса расширений топологических пространств — класс бикомпактно определенных и класс секвенциально определенных точных хаусдорфовых расширений данного топологического пространства.

Ivan Gotchev. **Superneighbourhood spaces and extensions of topological spaces.** The notion of superneighbourhood space is introduced. It is applied in the study of the extensions of topological spaces. The compactly determined and sequentially determined strict Hausdorff extensions of a given topological space are described in terms of special superneighbourhood structures.

1. INTRODUCTION

D. Doitchinov [1] introduced the notion of supertopological space comprising as special cases the notions of topological, proximity and uniform spaces.

A supertopology on a set X is a pair (M, \mathfrak{F}) , where M is a subset of the power set $P(X)$ and $\mathfrak{F} : M \rightarrow F(X)$ is a map from the set M to the set $F(X)$ of the filters on X , such that:

- (a) M contains $J(X) = \{\emptyset \mid \cup \mid x \mid, x \in X\}$ and $\mathfrak{F} \mid \emptyset \mid = P(X)$;
- (b) If $A \in M$ and $U \in \mathfrak{F}(A)$, then $A \subset U$;
- (c) If $A \in M$ and $U \in \mathfrak{F}(A)$, then there exists a $V \in \mathfrak{F}(A)$, such that $U \in \mathfrak{F}(B)$ for each $B \in M$, $B \subset V$.

A set X , equipped with a supertopology, is called supertopological space.

Obviously, in the case $M = J(X)$ the notion of supertopological space coincides with the notion of topological space. Thus every supertopological space is a topological space.

An extension (Y, φ) of a topological space X , where Y is a topological space and $\varphi : X \rightarrow Y$ is a dense embedding, is called compactly determined extension of X , if for any $y \in Y$ there exists a set $A \subset X$ such that $y \in \overline{\varphi(A)}$ and $\overline{\varphi(A)}$ is compact.

It is obvious that all locally compact extensions and all first countable extensions of a space X are compactly determined extensions of X .

D. Doitchinov [2] used supertopologies for the construction of compactly determined extensions of a given Hausdorff topological space X . In the case of regular space X , he described in terms of supertopologies on X all regular compactly determined extensions of X .

A. Tozzi and O. Wyler [3] changed slightly the definition of supertopological space and showed that the category of these spaces is a topological category in the sense of [4].

In this paper we introduce the notion of a superneighbourhood structure and following the techniques and methods in [2] we obtain in Section 3 a description of all strict Hausdorff compactly determined extensions of a given Hausdorff topological space X . An extension (Y, φ) of a topological space X is strict if for every $y \in Y \setminus \varphi(X)$ and for every open set U with $y \in U$ there is an open set V , such that $y \in V \subset U$, which contains every open set W in Y with $V \cap \varphi(X) = W \cap \varphi(X)$. (For other equivalent definition see [5]). Since every regular extension is strict, we obtain in this way an extension of the results of D. Doitchinov. The role of the new notion of superneighbourhood structures becomes decisive in the case of non-regular extensions, which cannot be described by supertopologies as in [2].

In Section 2 we give all necessary definitions and some properties of the category of superneighbourhood spaces in the spirit of [3].

An extension (Y, φ) of a topological space X is called sequentially determined extension of X if for every $y \in Y$ there is a sequence $\{x_i\}_{i=1}^{\infty}$ in X such that $\lim_{i \rightarrow \infty} \varphi(x_i) = y$. Clearly, every Hausdorff sequentially determined extension of a

Hausdorff topological space X is a compactly determined extension of X .

In Section 4 we show that there is a bijection between a class of special superneighbourhood structures (supertopologies) on a given Hausdorff (regular) space X and all strict Hausdorff (regular) sequentially determined extensions of X . In fact, the construction given in this section can be slightly modified to give all Hausdorff sequentially determined extensions of a given space X . This is in contrast with [2], where all Hausdorff extensions, defined by means of supertopologies, are strict.

In this paper we will not discuss the question if a given space X has a non-trivial sequentially (compactly) determined extension. Spaces, which have no proper sequentially (compactly) determined extensions within a certain class \mathcal{P} of topological spaces, are studied in [6], [7] and [8]. This provides a new approach to the classical notion of \mathcal{P} -closed spaces (see [9] and [10]).

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2. SUPERNEIGHBOURHOOD SPACES

2.1. Definitions. A neighbourhood structure on a set X is a pair (M, \mathfrak{F}) , where M is a subset of the power set $\mathcal{P}(X)$ and $\mathfrak{F} : M \rightarrow \mathcal{F}(X)$ is a map from M to the set $\mathcal{F}(X)$ of all filters on X , such that:

- (1) M contains $\mathcal{J}(X) = \{ \emptyset \cup \{x\} \mid x \in X \}$ and $\mathfrak{F}(\emptyset) = \mathcal{P}(X)$;
- (2) If $A \in M$ and $A' \subset A$, then $A' \in M$;
- (3) If $A \in M$ and $U \in \mathfrak{F}(A)$, then $A \subset U$;
- (4) If $A \in M$, $U \in \mathfrak{F}(A)$ and $A' \subset A$, then $U \in \mathfrak{F}(A')$.

We define a superneighbourhood structure (briefly—SN-structure) on a set X as a neighbourhood structure (M, ϑ) on X which satisfies the following condition:

(5) If $A, B \in M$ and $U \in \vartheta(A)$, then there is a set $V \in \vartheta(A)$ such that $U \in \vartheta(V \cap B)$.

A supertopology on a set X is a neighbourhood structure (M, ϑ) on X , which satisfies the following condition:

(6) If $A \in M$ and $U \in \vartheta(A)$, then there is a set $V \in \vartheta(A)$ such that always $U \in \vartheta(B)$ if $B \in M$ and $B \subset V$.

It is easily seen that (4) is a consequence of (3) and (5), and (5) is a consequence of (3) and (6). This means that every supertopology on X is a SN-structure on X . If (M, ϑ) is a SN-structure (neighbourhood structure) on X then the triple (X, M, ϑ) , or briefly X , is called a superneighbourhood space (neighbourhood space).

If (M, ϑ) is an SN-structure on a set X and if $A \subset X$, then the induced SN-structure (M_A, ϑ_A) on A is defined by

(7) $M_A = \{B \subset A \mid B \in M\}$,

(8) $\vartheta_A(B) = \{U \cap A \mid U \in \vartheta(B)\}$ for $B \in M_A$.

A neighbourhood structure (M, ϑ) on X is called additive if it satisfies the following condition:

(9) For sets A and B such that $A \cup B \in M$ and for sets $U \in \vartheta(A)$ and $V \in \vartheta(B)$ we have always $U \cup V \in \vartheta(A \cup B)$.

A continuous map $f: X \rightarrow Y$ of neighbourhood spaces X and Y with neighbourhood structures (M, ϑ) and (M', ϑ') respectively, is a map which satisfies the conditions:

(10) If $A \in M$, then $f(A) \in M'$,

(11) If $A \in M$ and $V \in \vartheta'(f(A))$, then $f^{-1}(V) \in \vartheta(A)$.

Let NBD be the category of neighbourhood spaces and their continuous maps, SN be the full subcategory of NBD with SN-spaces as objects, and let STOP be the full subcategory of SN with supertopological spaces as objects. Denote by ASTOP, ASN and ANBD respectively the categories of additive supertopological spaces, additive SN-spaces and of additive neighbourhood spaces with their continuous maps.

The following result is proved in [3].

2.2. Theorem. a) NBD is a topological category over sets and STOP is a topological subcategory of NBD;

b) The full embeddings $ASTOP \rightarrow STOP$ and $ANBD \rightarrow NBD$ are cotopological, preserving colimits and final sinks.

The following theorem can be proved in the same way.

2.3. Theorem. a) SN is a topological subcategory of NBD and STOP is a topological subcategory of SN.

b) The full embedding $ASN \rightarrow SN$ is cotopological, preserving colimits and final sinks.

2.4. b-SN-structures. A neighbourhood structure (M, ϑ) on a set X is called separated, provided the following condition is satisfied:

- (12) If $A, B \in \mathcal{M}$ and there exists an $U \in \mathcal{O}(A)$ with $U \cap B = \emptyset$ then there exist $V \in \mathcal{O}(A)$ and $W \in \mathcal{O}(B)$ with $V \cap W = \emptyset$.

If X is a topological space and $(\mathcal{M}, \mathcal{O})$ is a neighbourhood structure on X , then we say that $(\mathcal{M}, \mathcal{O})$ is compatible with the topology on X if for every $A \in \mathcal{M}$, $\mathcal{O}(A)$ is a filter on X with an open base and for every $x \in X$, $\mathcal{O}(x)$ is the filter of neighbourhoods of x on the topological space X .

Let X be a topological space and $(\mathcal{M}, \mathcal{O})$ be a compatible SN-structure on X . We say that $(\mathcal{M}, \mathcal{O})$ is a b -SN-structure on X if it is additive and separated. The b -supertopologies coincide with those, introduced in [2].

In this paper every SN-structure on a given topological space X will be compatible with the topology on X .

3. COMPACTLY DETERMINED EXTENSIONS

D. Doitchinov [2] showed that every b -supertopology on a given Hausdorff topological space X generates in a standard manner a Hausdorff compactly determined extension of X . Now we will show that this construction is also good for a SN-structure on X . The proofs, identical with proofs in [2], will be omitted.

3.1. Definition. Let $(\mathcal{M}, \mathcal{O})$ be a compatible b -SN-structure on a Hausdorff topological space X . A non-empty collection \mathcal{A} of non-empty subsets of X , belonging to \mathcal{M} , has the finite \mathcal{O} -intersection property if from $A_i \in \mathcal{A}$,

$U_i \in \mathcal{O}(A_i)$, $i = 1, \dots, k$ (k -arbitrary natural number) it follows $\bigcap_{i=1}^k U_i \neq \emptyset$.

It is easily seen that any collection, having the finite \mathcal{O} -intersection property, is contained in a maximal one.

Denote by X^* the set of all maximal collections in X , having the finite \mathcal{O} -intersection property with respect to the b -SN-structure $(\mathcal{M}, \mathcal{O})$. The elements of X^* will be denoted by ξ, η, ζ etc.

3.2. Lemma. Let $\xi \in X^*$ and $A \in \xi$. Then the following conditions hold:

- (a) If $B \in \mathcal{M}$ and $A \subset B$, then $B \in \xi$;
- (b) If $B \subset A$, then $B \in \xi$ or $A \setminus B \in \xi$;
- (c) If $B \in \xi$, and $V \in \mathcal{O}(B)$, then $A \cap V \neq \emptyset$ and $A \cap V \in \xi$.

3.3. Lemma. If $\xi \in X^*$, $A_i \in \xi$, $U_i \in \mathcal{O}(A_i)$, $i = 1, \dots, k$, then there is

$B \in \xi$ such that $\bigcap_{i=1}^k U_i \in \mathcal{O}(B)$.

Proof. Let $A_1 \in \xi$ and $A_2 \in \xi$, $U_1 \in \mathcal{O}(A_1)$ and $U_2 \in \mathcal{O}(A_2)$. Then by (5) there is a set $W \in \mathcal{O}(A_1)$ such that $U_1 \in \mathcal{O}(W \cap A_2)$. But by Lemma 3.2.(c) $B = W \cap A_2 \in \xi$ and hence $U_1 \in \mathcal{O}(B)$ and $U_2 \in \mathcal{O}(B)$. Thus $U_1 \cap U_2 \in \mathcal{O}(B)$.

3.4. Lemma. If $\xi \in X^*$, $A \in \mathcal{M}$, $A \neq \emptyset$ and if $A \notin \xi$, then there are $U \in \mathcal{O}(A)$, $B \in \xi$, $V \in \mathcal{O}(B)$ such that $U \cap V = \emptyset$.

Lemma 3.3. shows that

- (13) $F(\xi) = \{U \mid U \in \mathcal{O}(A), A \in \xi\}$

is a filter on X and Lemma 3.4 shows that if $\xi \neq \eta$ then there is $U \in F(\xi)$ and $V \in F(\eta)$ such that $U \cap V = \emptyset$.

Now we will introduce a topology on X^* . For any set $G \subset X^*$ define $\overline{G} = \{\xi \in X^* \mid \text{if } A \in \xi \text{ and } U \in \mathfrak{O}(A), \text{ then there exist } \eta \in G \text{ and } B \in \eta \text{ with } U \in \mathfrak{O}(B)\}$.

The operator $G \rightarrow \overline{G}$ is a Kuratowski closure operator. The space X^* will always be endowed with the topology induced by this closure operator. (For a proof in the case of a b -supertopological space X , see [2]).

For any open subset U of the space X let

$$(14) \quad \Omega_U = \{\xi \in X^* \mid U \in \mathfrak{O}(A) \text{ for some } A \in \xi\}.$$

The next four lemmas can be proved as in [2].

3.5. Lemma. a) For any two open subsets U and V of X , $\Omega_U \cap \Omega_V = \Omega_{U \cap V}$ holds;

b) The collection $\{\Omega_U \mid U \text{ is open in } X\}$ is a base for the topology on X^* .

3.6. Lemma. If $\xi, \eta \in X^*$ and $\xi \neq \eta$, then there exist $A \in \xi, B \in \eta, U \in \mathfrak{O}(A), V \in \mathfrak{O}(B)$ with $U \cap V = \emptyset$. In particular X^* is a Hausdorff space.

3.7. Lemma. For any $x \in X$ the collection

$$(15) \quad \alpha(x) = \{A \in \mathcal{M} \mid x \in U \text{ for every } U \in \mathfrak{O}(A)\}$$

is an element of the space X^* .

The equality (15) defines a mapping $\alpha: X \rightarrow X^*$.

3.8. Lemma. (X^*, α) is an extension of the topological space X .

3.9. Definition. The extension (X^*, α) of the space X will be called standardly generated by the b -SN-structure $(\mathcal{M}, \mathfrak{O})$ on X , and the mapping α -standard embedding of X into X^* .

The above defined extension can be described also by means of open filters as follows: for every $\xi \in X^*$ let $X' = \{F(\xi) \mid \xi \in X^*\}$, where $F(\xi)$ is defined in (13). By Lemma 3.7. $F(x) \in X'$ for every $x \in X$. Now we introduce in X' the strict topology. For an open base in X' we will take all $\theta_U = \{F(\xi) \mid U \in F(\xi)\}$, U is open in X . One can easily see that X^* and X' are homeomorphic extensions of X .

In the following three lemmas we prove that (X^*, α) is a compactly determined extension of X .

3.10. Lemma. Let $\xi \in X^*$ and $A \in \mathcal{M}, A \neq \emptyset$. Then $A \in \xi$ if and only if $\xi \in \alpha(A)$.

Proof. Let $A \in \xi$. We take $B \in \xi$ and $U \in \mathfrak{O}(B)$. By (5) there is $W \in \mathfrak{O}(B)$ such that $U \in \mathfrak{O}(A \cap W)$. But $A \neq \emptyset$ and by Lemma 3.2. (c) $A' = A \cap W \neq \emptyset$ and $A' \in \xi$. Let $x \in A'$. Then $\alpha(x) \in \alpha(A)$ and $A' \in \alpha(x)$. Now $U \in \mathfrak{O}(A')$ yields $\xi \in \alpha(A)$. For the inverse implication see [2].

3.11. Lemma. Let $A \in \mathcal{M}, A \neq \emptyset$ and let $(\mathcal{M}_A, \mathfrak{O}_A)$ be the b -SN-structure induced on A by $(\mathcal{M}, \mathfrak{O})$. If (A^*, α_A) is the standardly generated by $(\mathcal{M}_A, \mathfrak{O}_A)$ extension of the topological space A , then (A^*, α_A) and $(\alpha(A), \alpha/A)$ are equivalent extensions of A .

Proof. Let $\mathcal{M}_A = \mathcal{E}(A)$ and $\mathfrak{O}_A(B) = \{U \cap A \mid U \in \mathfrak{O}(B)\}$ for $B \in \mathcal{M}_A$. It is obvious that $(\mathcal{M}_A, \mathfrak{O}_A)$ is a b -SN-structure. Let $\xi \in A^*$. Then ξ has the finite \mathfrak{O} -intersection property in X . Then $\xi \subset \eta$ for some $\eta \in X^*$. Clearly $A \in \eta$. Let us assume that $\xi \subset \eta_1$ and $\xi \subset \eta_2$ for some $\eta_1 \neq \eta_2$ and $\eta_1, \eta_2 \in X^*$. Then by Lemma 3.6. there exist $C_1 \in \eta_1, U_1 \in \mathfrak{O}(C_1), C_2 \in \eta_2, U_2 \in \mathfrak{O}(C_2)$ such that $U_1 \cap U_2 = \emptyset$. By (5) there is $V_1 \in \mathfrak{O}(C_1)$ such that $U_1 \in \mathfrak{O}(V_1 \cap A)$ and by Lemma 3.2.(c) $V_1 \cap A \neq \emptyset$ and $V_1 \cap A \in \eta_1$. We shall see that $A \cap V_1 \in \xi$. But by Lemma 3.4.

it suffices to show that if $C \in \xi$, $U' \in \mathcal{D}_A(C)$ and $W' \in \mathcal{D}_A(V_1 \cap A)$, then $U' \cap W' \neq \emptyset$. We have $U' = U \cap A$, $W' = W \cap A$, $U \in \mathcal{D}(C)$ and $W = \mathcal{D}(V_1 \cap A)$. From $C \in \eta$, $V_1 \cap A \in \eta_1$ and by Lemma 3.3, it follows that $U \cap W \in \mathcal{D}(D)$ for some $D \in \eta_1$ and by Lemma 3.2.(c) $(U \cap W) \cap A \neq \emptyset$. Then $U' \cap W' \neq \emptyset$ and therefore $V_1 \cap A \in \xi$. But $U_1 \in \mathcal{D}(V_1 \cap A)$ and then $U_1 \cap A \in \mathcal{D}_A(V_1 \cap A)$. Analogously one can verify that there exists $V_2 \in \mathcal{D}(C_2)$ such that $V_2 \cap A \in \xi$ and $U_2 \cap A \in \mathcal{D}_A(V_2 \cap A)$. This implies $\emptyset \neq (U_1 \cap A) \cap (U_2 \cap A) \subset U_1 \cap U_2 = \emptyset$ — contradiction. Thus $\eta_1 = \eta_2$ and it is shown that for any $\xi \in A^*$ there exists only one point $\eta \in X^*$ with $\xi \subset \eta$. Let us note also that $\eta \in \overline{\alpha(A)}$ because $A \in \eta$.

Let us define a map $\lambda : A^* \rightarrow \overline{\alpha(A)}$ by means of the conditions $\xi \subset \lambda(\xi)$. We shall show that λ is a homeomorphism. The mapping λ is one-to-one. Indeed, assume that $\xi_1, \xi_2 \in A^*$, $\xi_1 \neq \xi_2$ and $\lambda(\xi_1) = \lambda(\xi_2)$, i. e. $\xi_1 \subset \eta$, $\xi_2 \subset \eta$ for some $\eta \in X^*$. By Lemma 3.6, there exist $B_1 \in \xi_1$, $B_2 \in \xi_2$ and $U_1 \in \mathcal{D}(B_1)$, $U_2 \in \mathcal{D}(B_2)$ such that $(U_1 \cap A) \cap (U_2 \cap A) = \emptyset$. But $B_1, B_2 \in \eta$ then $U_1 \cap U_2 \in \mathcal{D}(C)$ for some $C \in \eta$, hence $(U_1 \cap U_2) \cap A \neq \emptyset$. This is in contradiction with $\xi_1 \neq \xi_2$. So λ is one-to-one.

Now we prove the continuity of λ . Let $\Omega_U^A = \{\xi \in A^* \mid U \in \mathcal{D}_A(B) \text{ for some } B \in \xi\}$ for every open set $U \subset A$. Now assume that $\xi \in A^*$, $\eta = \lambda(\xi)$ and $\eta \in \Omega_U$, where U is an open subset of X . There exists $B \in \eta$ such that $U \in \mathcal{D}(B)$ satisfying $U \in \mathcal{D}(V \cap A)$. Let $W \in \mathcal{D}(B)$ be such that $V \in \mathcal{D}(W \cap A)$. As shown above, $W \cap A \in \xi$ then $V \cap A \in \mathcal{D}_A(W \cap A)$. Hence $\xi \in \Omega_{U \cap V}^A$. For every $\xi' \in \Omega_{U \cap V}^A$ there is C such that $V \cap A \in \mathcal{D}_A(C)$, then $C \subset V \cap A$. Now $U \in \mathcal{D}(V \cap A)$ yields $U \in \mathcal{D}(C)$. On the other hand $\xi' \subset \lambda(\xi')$, hence $C \in \lambda(\xi')$, $\lambda(\xi') \in \Omega_U$. Thus $\lambda(\Omega_{U \cap V}^A) \subset \Omega_U$. So the mapping λ is continuous and as the space A^* is compact, then λ is a homeomorphism.

It is easily seen that $\lambda(A^*) = \overline{\alpha(A)}$ and that $\lambda \circ \alpha_A = \alpha/A$. This completes the proof.

3.12. Corollary. For any $A \in \mathbf{M}$ the set $\overline{\alpha(A)}$ is compact.

Proof. Consider A with the induced b -SN-structure (M_A, \mathcal{D}_A) . Since $A \in M_A$, it can be proved in the same way as in [2] that A^* is compact.

Now the above lemma applies:

3.13. Corollary. Any b -SN-structure on a given Hausdorff space X generates (in a standard manner) a strict Hausdorff compactly determined extension (X^*, α) of X .

Let X be a Hausdorff space. Then the trivial extension (X, id_X) of X is compactly determined. The only Hausdorff extension of a compact space is the trivial one. This is why the next corollary follows from Corollary 3.12 and Corollary 3.13.

3.14. Corollary. If (M, \mathcal{D}) is a b -SN-structure on X , then (X^*, α) is the trivial extension of X if and only if for every $A \in M$, A is a compact subset of X .

3.15. Lemma. Let X be a Hausdorff topological space, A, B and U be subsets of X such that \overline{A} and \overline{B} are compact and U is an open set in X . If $\overline{A} \cap \overline{B} \neq \emptyset$ and $\overline{A} \subset U$, then there exists an open set V in X such that $\overline{A} \subset V$ and $\overline{A} \cap \overline{B} \subset U$.

Proof. If $\overline{B} \subset U$, then $V = U$. Let $\overline{B} \setminus U \neq \emptyset$ and V and W are open sets such that $\overline{B} \setminus U \subset W$, $\overline{A} \subset V$ and $V \cap W = \emptyset$. Hence $\overline{V \cap B} \subset \overline{V \cap \overline{B}} = \overline{B} \setminus W \subset \overline{B} \setminus W \subset \overline{B} \setminus (B \setminus U) \subset U$, and this completes the proof.

It follows by the previous lemma that in every Hausdorff topological space a compatible b -SN-structure can be defined in the following way.

3.16. Definition. Let X be a Hausdorff topological space and

$$(16) \quad \mathcal{M} = \{A \mid A \subset X, A \text{ is compact}\},$$

$$(17) \quad \mathcal{P}(A) = \{U \mid U \subset X \text{ and } \overline{A} \subset \text{Int}U\} \text{ for every } A \in \mathcal{M}.$$

The b -SN-structure $(\mathcal{M}, \mathcal{P})$ on X is called standard b -SN-structure on the space X .

There is an example of a Hausdorff space X in which the standard b -SN-structure is not a b -supertopology.

3.17. Lemma. Let (Y, φ) be a Hausdorff compactly determined extension of X and $y \in Y$. For every open set $U \subset Y$ such that $y \in U$, there exists $A \subset X$ such that $y \in \varphi(A) \subset U$ and $\varphi(A)$ is compact in Y .

Proof. Let $y \in U \subset Y$ and U be an open set in Y . There is $B \subset X$ such that $y \in \varphi(B)$ and $\varphi(B)$ is a compact subset of Y . If $\varphi(B) \subset U$, then $A = B$. If $\varphi(B) \not\subset U$, then $\varphi(B) \setminus U$ is a compact set and there exists an open set V such that $y \in V$ and $\overline{V} \cap (\varphi(B) \setminus U) = \emptyset$, or equivalently $V \cap \varphi(B) \subset U$. Now $A = \varphi^{-1}(B \cap V)$ is the required subset of X , since $\varphi(A) \cap \overline{V} \cap \varphi(B) \subset U$.

3.18. Definition. Let (Y, φ) be a Hausdorff compactly determined extension of the space X , then the b -SN-structure induced on X by the standard b -SN-structure of Y is called b -SN-structure on X , determined by (Y, φ) (here we identify X with $\varphi(X)$).

3.19. Proposition. Let (Y, φ) be a strict Hausdorff compactly determined extension of a given Hausdorff topological space X . Then the b -SN-structure $(\mathcal{M}^*, \mathcal{P}^*)$ on X determined by (Y, φ) generates (Y, φ) .

Proof. By definitions 3.16. and 3.18..

$$(18) \quad \mathcal{M}^* = \{A \subset X \mid \overline{\varphi(A)} \text{ is compact}\}$$

$$(19) \quad \mathcal{P}^*(A) = \{U \subset X \mid U = \varphi^{-1}(U^*), U^* \subset Y, \overline{\varphi(A)} \subset \text{Int}U\} \text{ for every } A \in \mathcal{M}^*.$$

We can assume that $X \subset Y$ and φ is the identical embedding. Then $\mathcal{M}^* = \{\overline{A} \subset X \mid A \text{ is compact}\}$ and $\mathcal{P}^*(A) = \{U \subset X \mid U = U^* \cap X, U^* \subset Y, \overline{A} \subset \text{Int}U^*\}$, where \overline{A} denotes the closure in Y .

One can easily prove, by means of Lemma 3.15, that $(\mathcal{M}^*, \mathcal{P}^*)$ is a compatible b -SN-structure on X . We will show that the extension (Y, φ) is equivalent to the standardly generated extension of X by $(\mathcal{M}^*, \mathcal{P}^*)$.

For every $y \in Y$ let

$$(20) \quad \lambda(y) = \{A \in \mathcal{M}^* \mid y \in \overline{A}\}.$$

It is obvious that $\lambda(y)$ is a non-empty collection of sets with the finite \mathcal{P}^* -intersection property in X . Therefore $\lambda(y) \subset \eta$ for some $\eta \in \mathcal{X}^*$. We shall see that $\lambda(y) = \eta$. Indeed, suppose that there exists $B \subset X$ such that $B \in \eta$ and $B \notin \lambda(y)$. Since $y \notin \overline{B}$ and \overline{B} is compact in Y , there exist open sets U^* and V^* in Y such that $y \in U^*$, $\overline{B} \subset V^*$ and $U^* \cap V^* = \emptyset$. Let $A \in \lambda(y)$. Then $y \in \overline{A} \cap U^* \subset \overline{A \cap U^*}$. Hence $A \cap U^* \in \lambda(y)$ and $A \cap U^* \in \eta$. On the other hand $V^* \cap X \in \mathcal{P}^*(B)$, and by Lemma 3.2.(c) $\emptyset \neq (A \cap U^*) \cap (V^* \cap X) \subset U^* \cap V^* = \emptyset$ — contradiction. Therefore $\lambda(y) = \eta$.

The equality (20) defines a mapping $\lambda : Y \rightarrow X^*$. In view of (18) we have $\lambda(x) = \alpha(x)$ for any $x \in X$. Now we will show that the mapping λ is a homeomorphism. Let $y_1 \neq y_2$, then there exist disjoint open sets U_1 and U_2 such that $y_1 \in U_1, y_2 \in U_2$, since Y is a Hausdorff space. By Lemma 3.17. and by the fact that (Y, φ) is a compactly determined extension of X it follows that there exist $\bar{A} \subset X$ and $\bar{B} \subset X$ such that $y_1 \in \bar{A} \subset U_1, y_2 \in \bar{B} \subset U_2$ and \bar{A}, \bar{B} are compact. Hence $y_2 \notin \bar{B}$ and $y_1 \notin \bar{A}$. Thus $\lambda(y_1) \neq \lambda(y_2)$ and λ is one-to-one.

To establish the continuity of λ we take a point $y_0 \in Y$ and consider an open set U in X such that $\lambda(y_0) \in \Omega_U$. Then $U \in \mathcal{P}^*(A)$ for some $A \in \lambda(y_0)$. This means that $U = U^* \cap X$ for some open U^* in Y and $y_0 \in A \subset U^*$. Now let $y \in U^*$. By Lemma 3.17. there exists $B \subset X$ such that $y \in B \subset U^*$ and B is compact in Y . Then $U \in \mathcal{P}^*(B)$, hence $\lambda(y) \in \Omega_U$. Thus $\lambda(U^*) \subset \Omega_U$, which proves the continuity of λ .

We show next, that λ is onto. Indeed, let $y \in X^*$. If $A \in \eta$ then $\eta \in \alpha(A) = \lambda(A)$. By the compactness of $\lambda(\bar{A})$, $\lambda(\bar{A}) \subset \lambda(A) \subset \lambda(\bar{A}) = \lambda(A)$, hence $\lambda(\bar{A}) = \lambda(A)$, consequently $\eta \in \lambda(\bar{A})$. It follows that $X^* = \lambda(Y)$.

So we proved the following result:

3.20 Proposition. Let (Y, φ) be a Hausdorff compactly determined extension of a given Hausdorff space X and $(\mathcal{M}^*, \mathcal{P}^*)$ be the b -SN-structure on X determined by (Y, φ) . If (X^*, α) is the standardly generated by $(\mathcal{M}^*, \mathcal{P}^*)$ compactly determined extension of X , then there is a bijective continuous mapping $\lambda : Y \rightarrow X^*$ such that $\lambda(\varphi(X)) = \alpha(X)$.

To complete the proof of Proposition 3.19. we shall see that $\lambda^{-1} : X^* \rightarrow Y$ is continuous. Let $\eta \in X^*$ and $\eta_0 = \lambda(y_0)$ for some $y_0 \in Y$ and let U^* be an open neighbourhood of y_0 in Y . Since (Y, φ) is a strict extension of X , there exists an open set $V^* \subset Y^*$ which contains every open set W^* in Y satisfying $W^* \cap X = V^* \cap X$ and $y_0 \in V^* \subset W^*$ holds. Now let $A \in \eta_0$. Then $y_0 \in \bar{A}$ and by Lemma 3.17 there exists $B \subset A$ such that $y_0 \in \bar{B} \subset V^*$ and \bar{B} is compact. Hence $B \in \eta_0$. On the other hand $B \in \mathcal{M}^*$ and if $V = V^* \cap X$ then $V \in \mathcal{P}^*(B)$. Therefore $\eta_0 \in \Omega_V$.

Now we shall show that $\Omega_V \subset \lambda(U^*)$. Indeed, let $\eta \in \Omega_V$ and $\eta \in \lambda(y)$ for some $y \in Y$. There exists a set $C \in \eta$ such that $V \in \mathcal{P}^*(C)$. This means that there is an open set W^* in Y such that $V = W^* \cap X$ and $y \in \bar{C} \subset W^*$. By the choice of V^* we obtain that $W^* \subset V^* \subset U^*$. Therefore $y \in U^*$. Hence $\Omega_U \subset \lambda(U^*)$ and the mapping λ^{-1} is continuous. This completes the proof.

In general, various b -SN-structures on a Hausdorff space X may generate the same extension (Y, φ) . Among them the b -SN-structure on X , determined by (Y, φ) , plays a special role.

3.21. Proposition. Let X be a Hausdorff topological space, (Y, φ) be a given strict Hausdorff compactly determined extension of X and $(\mathcal{M}^*, \mathcal{P}^*)$, be the b -SN-structure on X determined by (Y, φ) . If $(\mathcal{M}', \mathcal{P}')$ is another b -SN-structure on X generating the extension (Y, φ) , then $\mathcal{M}' \subset \mathcal{M}^*$ and $\mathcal{P}'(A) = \mathcal{P}^*(A)$ for each $A \in \mathcal{M}'$.

Proof. The extensions of X , standardly generated by $(\mathcal{M}^*, \mathcal{P}^*)$ and $(\mathcal{M}', \mathcal{P}')$, will be denoted by (X^*, α) and (X', α') , respectively. But (Y, φ) and (X^*, α) are equivalent extensions of X and then

$$(21) \mathcal{M}^* = \{A \subset X \mid \bar{\alpha(A)} \text{ is compact}\}.$$

(22) $\mathfrak{P}^*(A) = \{U \subset X \mid U = \alpha^{-1}(U^*), U^* \subset X^*, \overline{\alpha(A)} \subset \text{Int } U^*\}$ for any $A \in \mathcal{M}^*$.

On the other hand (X^*, α) and (X', α') are equivalent extensions of X . Then there exists a homeomorphism $\lambda: X' \rightarrow X^*$ with $\lambda(X') = X^*$ and $\lambda\alpha' = \alpha$. One can easily see that if $A \in \mathcal{M}'$ then

(23) $\overline{\alpha(A)} = \overline{\lambda(\alpha'(A))}$,

and hence $\mathcal{M}' \subset \mathcal{M}^*$ and $\mathfrak{P}'(A) \subset \mathfrak{P}^*(A)$. We shall prove that $\mathfrak{P}^*(A) \subset \mathfrak{P}'(A)$. Let $A \in \mathcal{M}'$ and $U \in \mathfrak{P}^*(A)$. Then $U = \alpha^{-1}(U^*)$, where U^* is an open set in X^* and $\overline{\alpha(A)} \subset U^*$. Let $U' = \lambda^{-1}(U^*)$. By (23) we have $\overline{\alpha'(A)} = \overline{\lambda^{-1}(\alpha(A))} \subset \lambda^{-1}(U^*) = U'$. For every $\eta \in \overline{\alpha'(A)}$ there exists an open set W' in X such that $\eta \in \Omega'_{W'} \subset U'$. Hence $W' \in \mathfrak{P}(B)$ for some $B \in \eta$. By Lemma 3.10 $A \in \eta$ and by Lemma 3.2.(c) and by (5) there exists $W'' \in \mathfrak{P}(B)$ such that $W' \in \mathfrak{P}(W'' \cap A)$ and $W'' \cap A \neq \emptyset$. By the compactness of $\overline{\alpha'(A)}$ there exist finitely many open sets

W'_1, \dots, W'_k and $W''_1, W''_2, \dots, W''_k$ in X such that $\alpha'(A) \subset \bigcup_{i=1}^k \Omega'_{W'_i}$

$\Omega'_{W'_i} \subset U'$ and $W'_i \in \mathfrak{P}(W''_i \cap A)$, $i = 1, 2, \dots, k$. Then $A \subset \bigcup_{i=1}^k \alpha'^{-1}(\Omega'_{W'_i}) = \bigcup_{i=1}^k W''_i$

and consequently $A = \bigcup_{i=1}^k (A \cap W''_i)$. From $W'_i \in \mathfrak{P}(A \cap W''_i)$ for $i = 1, 2, \dots, k$

and from the additivity of the SN-structure $(\mathcal{M}', \mathfrak{P}')$ it follows that $\bigcup_{i=1}^k W''_i \in \mathfrak{P}'(A)$.

But $\bigcup_{i=1}^k W''_i = \bigcup_{i=1}^k \alpha'^{-1}(\Omega'_{W'_i}) = \alpha'^{-1}(\bigcup_{i=1}^k \Omega'_{W'_i}) \subset \alpha'^{-1}(U') = \alpha^{-1}(\lambda(U')) = \alpha^{-1}(U^*)$

$= U$. Hence $\bigcup_{i=1}^k W''_i \subset U$, and therefore $U \in \mathfrak{P}'(A)$. This means that $\mathfrak{P}^*(A) \subset \mathfrak{P}'(A)$,

and this completes the proof.

Corollary 3.14. and Propositions 3.19., 3.20. and 3.21. yield the following result
3.22. Theorem. Any b -SN-structure on a given Hausdorff space X generates (in a standard manner) a strict Hausdorff compactly determined extension of X . On the other hand, every strict Hausdorff compactly determined extension of X is generated by some b -SN-structure on X . Among the b -SN-structures on X , generating a given strict Hausdorff compactly determined extension (Y, φ) of X , there exists one $(\mathcal{M}^*, \mathfrak{P}^*)$ with the following property: if the b -SN-structure $(\mathcal{M}, \mathfrak{P})$ on X generates (Y, φ) , then $\mathcal{M} \subset \mathcal{M}^*$ and $\mathfrak{P}(A) = \mathfrak{P}^*(A)$ for each $A \in \mathcal{M}$.

This theorem generalizes Theorem 1 from [2].

Let $(\mathcal{M}, \mathfrak{P})$ be a b -SN-structure on a given Hausdorff space X . We say that $(\mathcal{M}, \mathfrak{P})$ is a maximal b -SN-structure on X if the following condition is satisfied:

(24) If $A \subset X$ and for every filter F on A there exists $\xi \in X^*$ such that $F \cup F(\xi)$ is a filter base on X then $A \in \mathcal{M}$.

Let us note that in (24) $F \cup F(\xi)$ is a filter if and only if ξ is a cluster point of F in X^* . This is why the b -SN-structure of a Hausdorff space X determined by any compactly determined Hausdorff extension (Y, φ) of X is maximal.

3.23. Proposition. Let X be a Hausdorff space, (\mathcal{M}, ϑ) be a maximal b -SN-structure on X and let (X^*, α) be the standardly determined extension of X . Then (\mathcal{M}, ϑ) coincides with the b -SN-structure on X determined by (X^*, α) .

Proof. By Proposition 3.21, it follows that $\mathcal{M} \subset \mathcal{M}^*$ and $\vartheta(A) = \vartheta^*(A)$ for every $A \in \mathcal{M}$. We shall prove that $\mathcal{M}^* \subset \mathcal{M}$. Let $A \in \mathcal{M}^*$ and let F be a filter on A . Then $\varphi(F) = \{\varphi(F) \mid F \in \mathcal{F}\}$ is a filter on $\varphi(A)$. Let y be a cluster point of $\varphi(F)$ in Y and $\xi = \{BCX \mid y \in \varphi(B), B \in \mathcal{M}\}$. Then $F \cup F(\xi)$ is a filter on X and hence $A \in \mathcal{M}$. This completes the proof.

From Theorem 3.22, and Proposition 3.23, follows:

3.24. Theorem. There is bijective mapping from the set of all maximal b -SN-structures on a given Hausdorff topological space X to the set of all strict Hausdorff compactly determined extensions of X .

In particular the standard b -SN-structure on a given Hausdorff space X is maximal, it corresponds to the trivial extension of X .

Let us observe here that it is not too restrictive to consider only strict compactly determined extensions. In fact if (Y, φ) is any compactly determined Hausdorff extension of X , then for each compact set K in Y the extension $\varphi: K \cap X \rightarrow X$ is strict.

Now, let X be a regular space. One can easily see that the standard b -SN-structure on X is a b -supertopology on X . Hence if (Y, φ) is a regular compactly determined extension of a given space X and $(\mathcal{M}^*, \vartheta^*)$ is the b -SN-structure on X determined by (Y, φ) then $(\mathcal{M}^*, \vartheta^*)$ is a b -supertopology on X . It follows by Proposition 3.21 that if (Y, φ) is generated by some b -SN-structure (\mathcal{M}, ϑ) on X , then (\mathcal{M}, ϑ) will be a b -supertopology.

The following definition is given in [2].

A supertopology (\mathcal{M}, ϑ) on a set X is called strictly separated if it satisfies the condition:

(25) For any $A \in \mathcal{M}$ and any $U \in \vartheta(A)$ there exists a $V \in \vartheta(A)$ such that from $B \in \mathcal{M}$ and $B \cap U = \emptyset$ it follows $W \cap V = \emptyset$ for some $W \in \vartheta(B)$.

Clearly, any strictly separated supertopology is separated.

The following result, proved in [2], shows the relevance of this notion.

3.25. Proposition. Let X be a regular space and let (\mathcal{M}, ϑ) be a b -supertopology on X . The compactly determined extension of X , generated by (\mathcal{M}, ϑ) , is regular if and only if the supertopology (\mathcal{M}, ϑ) is strictly separated.

From Proposition 3.25 and Theorem 3.24 follows:

3.26. Theorem. There is a bijective mapping from the set of all strictly separated maximal b -supertopologies on a given regular space X to the set of all regular compactly determined extensions of X .

4. SEQUENTIALLY DETERMINED EXTENSIONS

4.1. Definition. Let X be a Hausdorff topological space and (\mathcal{M}, ϑ) be a b -SN-structure on X . We shall say that (\mathcal{M}, ϑ) is an s -SN-structure on X if the following conditions are satisfied:

(26) If $A \in \mathcal{M}$, then A is a countable set;

(27) If $A \in \mathcal{M}$, $B \subset A$ and B is infinite, then for every $U \in \vartheta(B)$ the set $A \setminus U$ is finite.

4.2. Proposition. For every s -SN-structure (M, ϑ) on a given Hausdorff topological space X the extension (X^*, α) , defined in Section 3, is a strict Hausdorff sequentially determined extension of X .

Proof. It follows by Corollary 3.13. that (X^*, α) is a strict Hausdorff compactly determined extension. We shall prove that (X^*, α) is a sequentially determined extension of X . Let $\xi \in X^* \setminus \alpha(X)$. Then by Lemma 3.10. it follows that every $A \in \xi$ is an infinite set. Let $A \in \xi$ and let Ω_U be an arbitrary basic neighbourhood of ξ . Then there exists $B \in \xi$ such that $U \in \vartheta(B)$. But by (5) and Lemma 3.2.(c) there exist $A' \subset A$ such that $U \in \vartheta(A')$. Thus by (27) $A \setminus U$ is finite and hence $\alpha(A) \setminus \Omega_U$ is finite. By (26) A is a countable set and this completes the proof.

In every Hausdorff topological space X there is a s -SN-structure defined by (28) $M = \{\overline{A} \subset X \mid \overline{A} \text{ is countable and compact}\}$,

(29) $\vartheta(A) = \{U \subset X \mid \overline{A} \subset \text{Int } U\}$ for every $A \in M$.

This s -SN-structure on X will be called standard s -SN-structure on X .

Let (Y, φ) be a sequentially determined extension of the Hausdorff space X . Then the s -SN-structure induced on X by the standard s -SN-structure on Y will be called standard s -SN-structure on X , determined by the extension (Y, φ) .

4.3. Proposition. Let (Y, φ) be a Hausdorff sequentially determined extension of a Hausdorff space X . Then the s -SN-structure (M^*, ϑ^*) on X , determined by (Y, φ) , generates (Y, φ) .

Proof. Follows easily from Proposition 3.19.

4.4. Proposition. Let (Y, φ) be a Hausdorff sequentially determined extension of a Hausdorff space X . Let also (M^*, ϑ^*) be the s -SN-structure on X , determined by (Y, φ) . If (X^*, α) is the standardly generated sequentially determined extension of X by (M^*, ϑ^*) , then there exists a continuous bijection $\lambda : Y \rightarrow X^*$, such that $\lambda(\varphi(X)) = \alpha(X)$.

Proof. Follows directly from Proposition 3.20 and Proposition 4.3.

4.5. Proposition. Let X be a Hausdorff topological space, (Y, φ) be a strict Hausdorff sequentially determined extension of X and (M^*, ϑ^*) be the s -SN-structure on X , determined by (Y, φ) . If (M', ϑ') is another s -SN-structure on X generating (Y, φ) then $M' \subset M^*$ and $\vartheta'(A) = \vartheta^*(A)$ for every $A \in M'$.

Proof. Follows from Proposition 3.21.

The next theorem follows from Propositions 4.3., 4.4. and 4.5.

4.6. Theorem. Any s -SN-structure on a given Hausdorff space X generates (in a standard manner) a strict Hausdorff sequentially determined extension of X . On the other hand, every strict Hausdorff sequentially determined extension of X is generated by some s -SN-structure on X . Among the s -SN-structures on X , generating a given strict Hausdorff sequentially determined extension (Y, φ) of X , there exists one (M^*, ϑ^*) with the following property: if the s -SN-structure (M, ϑ) on X generates (Y, φ) , then $M' \subset M^*$ and $\vartheta(A) = \vartheta^*(A)$ for each $A \in M$.

Let (M, ϑ) be an s -SN-structure on a given Hausdorff space X . We call that (M, ϑ) is a maximal s -SN-structure on X if the following condition is satisfied:

(30) A countable set A in X belongs to M whenever there exists $B \in M$ such that for every non-empty set $B' \subset B$, for which $B \setminus B'$ is finite, and for every $U \in \vartheta(B')$ the set $A \setminus U$ is finite.

4.7. Proposition. Let X be a Hausdorff space, (M, ϑ) be a maximal s -SN-structure on X , and let (X^*, α) be the standardly determined extension of X by (M, ϑ) . Then (M, ϑ) coincides with the s -SN-structure on X determined by (X^*, α) .

Proof. It follows by Proposition 4.5. that $M' \subset M^*$ and $\vartheta'(A) = \vartheta^*(A)$ for every $A \in M'$. We shall prove that $M^* \subset M$. We can assume that $X \subset Y$ and $\varphi : X \rightarrow Y$

is the identical embedding. Let $A \in \mathcal{M}^*$. If A is finite, then by the maximality of $(\mathcal{M}', \vartheta')$ we have $A \in \mathcal{M}'$. Let A is an infinite set. Then \bar{A} is a compact set and $\bar{A} \cap (Y \setminus X)$ is an only one point. Let $y = \bar{A} \cap (Y \setminus X)$. Then there exists $B \in \mathcal{M}'$, such that $y = B \cap (Y \setminus X)$. It is obvious that for every $B' \subset B$, such that $B \setminus B'$ is finite, and for every $U \in \vartheta(B')$ the set $A \setminus U$ is finite. Then $A \in \mathcal{M}'$, and this completes the proof.

The following result follows from Proposition 4.7

4.8. Theorem. There is a bijective mapping from the set of all maximal s -SN-structures on a given Hausdorff space X to the set of all strict Hausdorff sequentially determined extensions of X .

If X is a Hausdorff space, then the trivial extension of X is sequentially determined. If (\mathcal{M}, ϑ) is an s -SN-structure on X , then we may ask if the standardly generated extension of X by (\mathcal{M}, ϑ) is the trivial extension. The answer is given by the following lemma:

4.9. Lemma. Let X be a Hausdorff space and (\mathcal{M}, ϑ) be an s -SN-structure on X . Then the standardly determined extension (Y, φ) of X is the trivial extension if and only if for every $A \in \mathcal{M}$, \bar{A} is a compact subset of X .

Proof. It follows by Corollary 3.14.

4.10. Proposition. Let X be a regular space and (\mathcal{M}, ϑ) be an s -supertopology on X . The sequentially determined extension, generated by (\mathcal{M}, ϑ) , is regular if and only if (\mathcal{M}, ϑ) is strongly separated.

Proof. Follows directly from Proposition 3.25.

The following result comes from Theorem 4.8 and Proposition 4.10.

4.11. Theorem. There is a bijection between the set of all s -supertopologies on a given regular space X and the set of all regular sequentially determined extensions of X .

In the case of an s -SN-structure (\mathcal{M}, ϑ) on a Hausdorff space X , the topology of the standardly determined extension X^* can be modified in such a way that $X^* \setminus X$ be discrete. Denote by (kX, α) the extension of X defined in this way (the underlying sets of kX and X^* coincide). Clearly, (kX, α) is a Hausdorff sequentially determined extension of X , which is strict if and only if $kX \setminus X$ is finite. It is obvious that kX can be used to describe all Hausdorff sequentially determined extensions (Y, φ) of X such that $Y \setminus \varphi(X)$ is discrete. In the same way, choosing appropriate topologies on X^* between X^* and kX , we can describe all Hausdorff sequentially determined extensions of X .

The following example shows that this is not true for supertopologies.

4.12. Example. Let $Y = [0,1]$ with the topology, induced by the real line, and let $A = \{x_i\}_{i=1}^{\infty}$ be a sequence of positive rational numbers in Y , which converges to 0. We change the topology on Y in the following manner : for a closed base in Y we take the set A and all closed sets in Y . Now let X be the set of all irrational numbers in Y with the induced by Y topology. It is easily seen that the standard b -SN-structure (s -SN-structure) on Y is not a b -supertopology (s -supertopology). Clearly, Y is a Hausdorff sequentially determined extension of X but it cannot be described by supertopologies.

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