

## **TAKE-HOME EXAM 1**

**Solve the following problems showing all your work for full credit.**

**1.** Describe the natural domain of each function.

a) (2 pt.)  $f(x) = \frac{x}{x^2 - 4}$ ;

b) (2 pt.)  $g(x) = \sqrt{9 - x^2}$ ;

c) (2 pt.)  $f(x) = \frac{5x}{(x-3)\sqrt{x+5}}$ .

**2.** Which of the following functions are odd? Even? Neither even nor odd?

a) (1 pt.)  $f(x) = \frac{x^2 + 4}{3x}$

b) (1 pt.)  $g(x) = |\sin x| + \cos x$

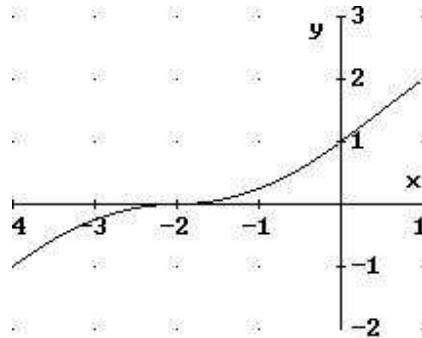
c) (1 pt.)  $h(x) = 2x^3 + 4\sin x$

**3.** A function  $f$  is given by  $f(x) = \begin{cases} 2x+3 & , \quad x < -2 \\ -3 & , \quad -2 \leq x < 4. \\ 5-x & , \quad 4 \leq x \end{cases}$

a) (3 pts.) Evaluate  $f(-3)$ ,  $f(1)$ , and  $f(6)$ .

b) (3 pts.) Make a hand-drawn graph of  $f(x)$ .

4. The graph of a function  $y = f(x)$  is shown below. No formula for  $f(x)$  is given.



- a) (2 pts.) Find the domain and range of  $f(x)$ .
- b) (3 pts.) Sketch the graph of the inverse function  $f^{-1}(x)$ .
- c) (3 pts.) Sketch the graph of the function  $g(x) = 3f(x) - 1$ .

5. (2 pts.) Let  $f(x) = x - \frac{1}{x}$  and  $g(x) = x^2 + 1$ . Find  $f \circ g$  and  $g \circ f$ .

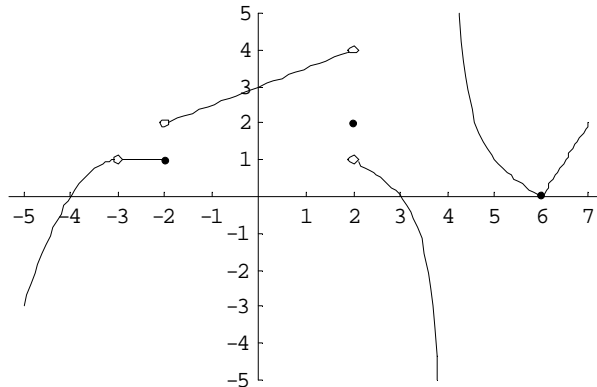
6. Calculate each of the following without using a calculator.

a) (1 pt.)  $\sin 570^\circ$

b) (1 pt.)  $\cos \frac{9\pi}{2}$

c) (1 pt.)  $\cos\left(\frac{-13\pi}{6}\right)$

7. (13 pts.) Consider the following graph of a function  $f$ .



Find the limit, if it exists.

- a)  $\lim_{x \rightarrow -3} f(x) =$       b)  $\lim_{x \rightarrow -2^+} f(x) =$       c)  $\lim_{x \rightarrow -2^-} f(x) =$   
d)  $\lim_{x \rightarrow 2^+} f(x) =$       e)  $\lim_{x \rightarrow 2^-} f(x) =$       f)  $\lim_{x \rightarrow 4^-} f(x) =$   
g)  $\lim_{x \rightarrow 4^+} f(x) =$       h)  $\lim_{x \rightarrow 6^+} f(x) =$       i) Find  $f(-3) =$   
j) Find  $f(2) =$       k) Is  $f(x)$  continuous at  $-3$ ?  
l) Is  $f(x)$  continuous at  $2$ ?      m) Is  $f(x)$  continuous at  $6$ ?

8. Find the limit if it exists.

- a) (1 pt.)  $\lim_{x \rightarrow 0} \left( \frac{3x^2 + 1}{2x + 5} \right) =$   
b) (1 pt.)  $\lim_{x \rightarrow 1} \sin \frac{\pi x}{2} =$   
c) (2 pts.)  $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3} =$   
d) (3 pts.)  $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^2 + x - 2} =$   
e) (3 pts.)  $\lim_{x \rightarrow 1} \frac{1 - x}{1 - \sqrt{x}} =$   
f) (3 pts.)  $\lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x} =$

g) (2 pts.)  $\lim_{\theta \rightarrow 0} \frac{\tan 5\theta}{\sin 2\theta} =$

h) (2 pts.)  $\lim_{\theta \rightarrow 0} \frac{\cos \theta \tan \theta}{\theta} =$

i) (1 pt.)  $\lim_{x \rightarrow \infty} \frac{x^2}{(x-2)(x-3)} =$

j) (1 pt.)  $\lim_{x \rightarrow \infty} \frac{x^2}{5-x^3} =$

k) (1 pt.)  $\lim_{x \rightarrow \infty} \frac{x^3}{(x-1)(x+3)} =$

l) (1 pt.)  $\lim_{x \rightarrow -3^+} \frac{x^2 + 9}{x + 3} =$

m) (1 pt.)  $\lim_{x \rightarrow 4^-} \frac{x^2}{(x-1)(x-4)} =$

9. Find the limit L. Then use the  $\varepsilon - \delta$  definition to prove that the limit is L.

a) (7 pts.)  $\lim_{x \rightarrow -3} (2x + 5)$

b) (7 pts.)  $\lim_{x \rightarrow 1} (2x^2 + 1)$

10. (6 pts.) Find the vertical and horizontal asymptotes (if any) of the function

$$g(x) = \frac{3}{9-x^2} \text{ and sketch its graph.}$$

11. (4 pts.) Explain why the function  $f(x) = x^3 + 3x - 2$  has a zero in the interval  $[0, 1]$ .

12. (3 pts.) Find the constant  $a$  such that the function  $f(x) = \begin{cases} x^3, & x \leq 2 \\ ax^2, & x > 2 \end{cases}$  is continuous on the entire real line.

13. (4 pts.) Sketch the graph of a function that has domain  $[0,6]$  and is continuous on  $[0,2]$  and  $(2,6]$  but is not continuous on  $[0,6]$ .

14. (2 pts.) At what points, if any, is the function  $f(x)$  discontinuous? Why?

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3-x & \text{if } 0 \leq x < 3 \\ (x-3)^2 & \text{if } x > 3 \end{cases}$$

15. (4 pts.) The function  $f(x) = \frac{x^4 + 2x^2 - 3}{x+1}$  is not defined at a certain point. How should it be defined in order to make it continuous at that point?