

2. (10 pts.) Prove that if $f : X \rightarrow Y$, then f is continuous if and only if $f^{-1}(B)$ is open in X for each member B of some base for Y . Prove the same result with “base” replaced by “subbase”.
3. (5 pts.) Prove that if a and b are real numbers with $a < b$, then (a,b) is homeomorphic to R .
4. The collection $\tau_s = \{(a,b) : a < b\}$ is a base for a topology on R . The space (R, τ_s) is denoted by S and is called the Sorgenfrey line.
- a) (5 pts.) Prove that $S \times S$ is separable.
- b) (10 pts.) Find a subspace of $S \times S$ that is not separable, hence “separable” is not a hereditary property.
- c) (10 pts.) Prove that S is Lindelöf.

d) (10 pts.) Prove that $S \times S$ is not Lindelöf.

e) (10 pts.) Prove that S is zero-dimensional.

f) (5 pts.) Prove that S is normal.

5. Prove that

a) (5 pts.) Every subspace of a regular space is regular.

b) (5 pts.) If X_n is regular for each $n \in \omega$, then $\prod_{n=0}^{\infty} X_n$ is regular.