Section 10.2

1. a. $b > 1$
   b. $0 < b < 1$
3. The initial value $y_0$ is the coefficient in the growth function, $y = y_0 b^x$.
   a. 1
   b. 5
   c. 4
   d. $\frac{1}{2}$

5. $Y_1 = 2(3)^x$, $Y_2 = 2(3)^{-x}$
   Windows may vary: $X_{\text{min}} = -4.7$, $X_{\text{max}} = 4.7$, $Y_{\text{min}} = -5$, $Y_{\text{max}} = 10$

The graph of $y = 2(3)^{-x}$ is the mirror image of the graph of $y = 2(3)^x$ with the $y$-axis as the mirror.

7. Half-life gives a quick method to compare rates of decay. Half-life is the amount of time it takes for a substance to decay to one-half of its initial amount.

9. a. For equal time periods the output decreases by the same factor. In this case, after every 13 days the weight is halved.
   b. $y_0 = 1$: the initial value is the weight of the substance at time $= 0$.
   c. $b = \frac{1}{2}$: the decay factor is the ratio of successive outputs.
   d. $k = \frac{1}{10}$: the time scale is the number of days to halve.
   e. Substituting for $y_0$, $b$, and $k$ into the general exponential function $y = y_0 b^{kx}$, we have

11. a. $y_0 = 90$: the initial value is $\$90$.
   b. $\frac{90}{2} = \$45$

c. From the table of values, an estimate for the half-life of the value of a video game is between 1 and 2 years.

d. $Y_1 = 45$, $Y_2 = 90(2/3)^x$
   Windows may vary: $X_{\text{min}} = 0$, $X_{\text{max}} = 4.7$, $Y_{\text{min}} = 0$, $Y_{\text{max}} = 100$

From the graph, a better approximation for the half-life is 1.7 years.

13. a. $y_0 = 60$ The initial amount of thorium 234
   Thorium 234 decays 25% = $0.25 = \frac{1}{4}$ in each time period so $1 - \frac{1}{4} = \frac{3}{4}$ remains.
   b. $b = \frac{3}{4}$ The decay factor
   c. $k = \frac{1}{10}$ The time scale for decay
   d. Substituting for $y_0$, $b$, and $k$ into $y_0 b^{kx}$

15. The machine is worth half of its initial value in about $\frac{72}{14}$ = 5 years.

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Section 10.3

17. The population doubles in about \( \frac{72}{7} \approx 10.3 \) years.

25. \( x^2 - x - 20 \)
    The factor pair of \(-20x^2\) that sums to \( -1x \) is \( 4x \) and \( -5x \).
    \[(x^2 + 4x) + (-5x - 20)\]
    \[x(x + 4) - 5(x + 4)\]
    or \((x - 5)(x + 4)\)

Skills and Review 10.2

19. a. \( y \) varies directly as the square of \( x \)
    \[ y = ax^2 \]
    \[ \frac{5}{2} = a(1)^2 \] Substitute 1 for \( x \) and \( \frac{5}{2} \) for \( y \)
    \[ \frac{5}{2} = a \] Solve for \( a \)

b. \( y = ax^2 \)
    Substitute \( \frac{5}{2} \) for \( a \) gives the specific variation function.
    \[ y = \frac{5}{2} x^2 \]
    \[ \frac{5}{2} = \frac{5}{2}(3)^2 \] Substitute 3 for \( x \)
    \[ \frac{5}{2} = \frac{5}{2} \cdot 9 \]
    \[ \frac{5}{2} = \frac{45}{2} \]
    \[ \frac{45}{2} = 22.5 \]

c. \( y = \frac{5}{2} x^2 \)
    Substitute 67.6 for \( y \)
    \[ 67.6 = \frac{5}{2} x^2 \]
    \[ 27.04 = x^2 \]
    \[ x = 5.2 \]

21. \( 3^4 = 81 \) so \( 81^{1/4} = 3 \)

23. \( f(x) = 2x - 3, g(x) = x + 1 \)

a. \( g(-5) = -5 + 1 \)
    \[ g(5) = -4 \]
    Substitute \(-4\) for \( g(5) \)
    \( f(g(-5)) = f(-4) \)
    \[ f(-4) = 2(-4) - 3 \]
    \[ = -8 - 3 \]
    \[ = -11 \]

b. Substitute \( x + 1 \) for \( g(x) \) and substitute \( x + 1 \) for \( x \) into \( f(x) = 2x - 3 \)
    \( f(g(x)) = f(x + 1) \)
    \[ f(x + 1) = 2(x + 1) - 3 \]
    \[ = 2x + 2 - 3 \]
    \[ = 2x - 1 \]

c. Substituting \( 2x - 3 \) for \( f(x) \) and substitute \( 2x - 3 \) for \( x \) into \( g(x) = x + 1 \)
    \( g(f(x)) = g(2x - 3) \)
    \[ g(2x - 3) = (2x - 3) + 1 \]
    \[ = 2x - 2 \]