Section 11.3

1. a. Windows may vary; Xmin = 0, Xmax = 9.4, Ymin = 0, Ymax = 1.
   b. The intersection of Y1 and Y2 is approximately (6, 0.66666667). This indicates that the solution to the equation is \( x = 6 \), which agrees with the result in Example 3.

3. \( Y_1 = \frac{1}{(X - 2)} + 3; \ Y_2 = 7 \)
   Windows may vary; Xmin = -9.4, Xmax = 9.4, Ymin = -10, Ymax = 10
   After pressing 2nd CALC intersect, you may need to move the cursor right in order to start on the right branch of the hyperbola.

   The solution is \( x = 2.25 \).
5. Let \( x \) be represent the time (in hrs) for the intake pipe to fill the pool working alone. Then \( x + 3 \) represents the time (in hours) for the outtake pipe to drain the pool working alone. Following the same method of reasoning and solution in Example 4, in this problem we have

\[
\begin{align*}
\frac{1}{x} & = \frac{1}{x + 3} \\
\frac{1 \cdot (x + 3)}{x \cdot (x + 3)} & = \frac{1 \cdot x}{x \cdot x} \\
\frac{x + 3}{x} & = \frac{x}{x + 3} \\
\frac{3}{x^2 + 3x} & = \frac{1}{x} \\
x^2 + 3x & = 18 \\
x + 6 & = 0 \quad \text{or} \quad x - 3 = 0 \\
x & = -6 \quad \text{or} \quad x = 3 \\
\end{align*}
\]

Since \( x \) represents time, we ignore the negative solution. Thus it takes 3 hours to fill the pool with the outtake pipe shut.

7. \( 6 = \frac{20}{x - 2} \)

\[
\begin{align*}
6(x - 2) & = 20 \\
6x - 12 & = 20 \\
6x & = 32 \\
x & = \frac{32}{6} = \frac{16}{3} \\
\end{align*}
\]

9. \( \frac{3x}{x - 1} + \frac{8(x - 1)}{x} = 3 \)

\[
\begin{align*}
\frac{3x \cdot x}{x(x - 1)} + \frac{8 \cdot (x - 1) \cdot x}{x \cdot (x - 1)} & = 3 \\
\frac{3x^2 + 8x - 8}{x(x - 1)} & = 3 \\
\frac{3x^2 + 8x - 8}{x^2 - x} & = 3 \\
\frac{11x - 8}{x^2} & = 3 \\
11x - 8 & = 3(x^2 - x) \\
11x - 8 & = 3x^2 - 3x \\
0 & = 3x^2 - 14x + 8 \\
0 & = (3x - 2)(x - 4) \\
3x - 2 & = 0 \quad \text{or} \quad x - 4 = 0 \\
x & = 2 \quad \text{or} \quad x = 4 \\
\end{align*}
\]

11. Let \( x \) represent the time (in hours) that it takes for Joan and her partner to detail a car working together. Recognize that the time should be less than 4 hrs (the time it takes Joan to do the job by herself) because she has her partner’s help. Construct a table as follows:

<table>
<thead>
<tr>
<th>Work</th>
<th>Time</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joan</td>
<td>1 car</td>
<td>4 hrs</td>
</tr>
<tr>
<td>Partner</td>
<td>1 car</td>
<td>6 hrs</td>
</tr>
<tr>
<td>Together</td>
<td>1 car</td>
<td>( x ) hrs</td>
</tr>
</tbody>
</table>

Since Joan and her partner are working together, we add their work rates:

\[
\frac{1}{x} + \frac{1}{x + 3} = \frac{1}{4} \\
\frac{1 \cdot (x + 3)}{x \cdot (x + 3)} + \frac{1 \cdot x}{x \cdot x} = \frac{1}{4} \\
\frac{x + 3 + x}{x(x + 3)} = \frac{1}{4} \\
\frac{3}{x^2 + 3x} = \frac{1}{4} \\
x^2 + 3x = 12 \\
x^2 + 3x - 12 = 0 \\
x = \frac{3 \pm \sqrt{9 + 4 \cdot 12}}{2} \\
x = \frac{3 \pm \sqrt{57}}{2} \\
x = 4 \quad \text{or} \quad x = -3 \\
\]

13. Let \( x \) be the speed of the river current (in m/s).

a. Since the team can row 80 m/s in still water, then they can row \((80 - x)\) m/s going upstream.

\[
\text{distance} = \text{rate} \times \text{time} \\
3000 = (80 - x) \times 50 \\
60 = 80 - x \\
-20 = -x \\
20 = x \\
The speed of the river current is 20 m/s.
\]

b. The team rows at a rate of 80 m/s + 20 m/s = 100 m/s downstream. The total traveling distance is still 3000 m.

\[
\text{time} = \frac{\text{distance}}{\text{rate}} = \frac{3000}{100} = 30 \\
The rowing team travels downstream in 30 seconds.
15. a. Windows may vary: Xmin = 9.4, Xmax = 9.4, Ymin = 10, Ymax = 15

No, the graph of $Y_1 = (7X + 3)/X$ has a horizontal asymptote at $Y = 7$. As $X$ assumes very large values (negative or positive) the value of $Y$ will approach 7, but never equal 7.

b. \[ \frac{7x + 3}{x} = 7 \]
\[ 7x + 3 = 7x \]
\[ 3 = 0 \]
But 3 \neq 0

c. Either by trying to solve the equation \[ \frac{7x + 3}{x} = 7 \] by graphing or with algebra, we find the equation has no solution.

Skills and Review 11.3

17. a. \[ y_1 = \frac{x}{x + 1} \]

b. Vertical asymptote: $y$-axis ($x = 0$); horizontal asymptote: $y = 1$

19. \[ \frac{x + 3x^2}{x - 7} = \frac{x + 7 - x}{x - 7} = \frac{7x - x^2}{3x^2(x - 7)} = \frac{-1(x + 7)}{3x^2(x - 7)} = \frac{-1}{3x} \]

21. \[ \log(10^{-2}) = -2 \]

23. \[ (x^{3/5})^{20} = x^{20 \cdot \frac{3}{5}} = x^{12} = \left(\frac{1}{x}\right)^{12} \]

25. a. \[ x^2 + 5x = -6 \]
\[ x^2 + 5x + 6 = 0 \]
\[ (x + 3)(x + 2) = 0 \]
\[ x + 3 = 0 \quad \text{or} \quad x + 2 = 0 \]
\[ x = -3 \quad \text{or} \quad x = -2 \]

b. \[ 4(10^x) = 400 \]
\[ 10^x = 100 \]
\[ \log(10^x) = \log(10^2) \]
\[ x = 2 \]

c. \[ x = \log(10) = \log(10^2) = 1 \]

d. \[ \frac{3}{5} x^4 = \frac{48}{5} \]
\[ x^4 = \frac{48}{5} \cdot \frac{5}{3} \]
\[ x^4 = 16 \]
\[ (x^4)^{1/4} = \pm(16)^{1/4} \]
\[ x = \pm(2^{4})^{1/4} \]
\[ x = \pm 2 \]

e. \[ \frac{1}{2} \quad \frac{2}{x} \quad \frac{3}{x} \quad \frac{4}{x} \quad \frac{5}{x} \]
\[ 1 \cdot x + 2 \cdot \frac{x}{x} = \frac{1}{x} \]
\[ 2 \cdot \frac{x}{x} + 2 \cdot \frac{x}{x} = \frac{1}{x} \]
\[ x + 4 = \frac{1}{x} \]
\[ 2 \cdot \frac{x}{x} = \frac{1}{x} \]
\[ x(x + 4) = 2x \]
\[ x^2 + 4x = 2x \]
\[ x^2 + 2x = 0 \]
\[ x(x + 2) = 0 \]
\[ x = 0 \quad \text{or} \quad x + 2 = 0 \]
\[ x = 0 \quad \text{or} \quad x = -2 \]
\[ x = 0 \quad \text{is an extraneous solution because when 0 is substituted for } x \quad \text{into the original equation it gives undefined terms. Therefore the only solution is } x = -2. \]