Chapter 12: Inequalities

Section 12.1

1. a. Answers may vary; $x = 1, 2.5, 3, \frac{5}{4}$.
   Melissa crosses any location between Avenue 2 and Avenue 6.
b. Yes. The phrase never gets farther than 4 blocks from her house means we include the endpoints 2 and 6.

3. a. Answers may vary; 250 minutes and 300 minutes.
b. $x = 200$ minutes is not a solution because the phrase more than means we exclude the endpoint 200.
   $x = 400$ is a solution because the phrase no more than means we include the endpoint 400.

5. a. $x < 4$
b. $x > 4$
c. $x \leq 4$
d. $x \geq 4$
e. $x \leq 4$; at most means all numbers up to and including the endpoint.
f. $x \geq 4$; at least means all numbers above and including the endpoint.
g. $x \leq 4$; no more than means the same as at most.
h. $x \geq 4$; no less than means the same as at least.
i. $x > 4$; above means all numbers greater than and excluding the endpoint.
j. $x < 4$; below means all numbers less than and excluding the endpoint.

7. a. Because the two inequalities are connected by the word and, the solution set contains only those values that are solutions to both inequalities.

   \[
   \begin{align*}
   & -15 \leq 3x \\
   & 3x \leq 5 \\
   \end{align*}
   \]

b. Because the two inequalities are connected by the word or, the solution set contains those values that are solutions to either one or both inequalities.

   \[
   \begin{align*}
   & -3 \leq x \\
   & x \leq 5 \\
   \end{align*}
   \]

9. Multiplying or dividing an inequality by a negative number reverses the inequality symbol.
a. $-3x \leq 18$
   $x \geq 6$ Dividing by $-3$ reverses $\leq$ to $\geq$

b. $\frac{1}{2}x > -7$
   $x < 14$ Multiplying by $\frac{1}{2}$ reverses $> >$ to $<$

11. $4 + x \geq 2 -(7x + 1)$
   $4 + x \geq 2 -7x -1$
   $4 + x \geq 1 -7x$
   $4 + 8x \geq 1$
   $8x \geq -3$
   $x \geq -\frac{3}{8}$

   Note: although the solution is negative we do not reverse the inequality symbol unless we divide or multiply by a negative number.

13. $5 \leq 20 -3x$ and $20 -3x < 12$
   $15 \leq -3x$ and $-3x < -8$
   $5 \leq x$ and $x > \frac{8}{3}$
   $x \leq 5$ and $x > \frac{8}{3}$
   $x \leq 5$ is the same as $5 \geq x$
   Writing in set builder notation gives
   \[
   \{ x | \frac{8}{3} < x \leq 5 \}
   \]

15. a. $40 + L + 40 + L = 80 + 2L$
b. $200 \leq 80 + 2L$ and $80 + 2L \leq 300$
c. and; we want the perimeter to be at least 200 m and no more than 300 m.
d. $120 \leq 2L$ and $2L \leq 220$
   $60 \leq L$ and $L \leq 110$
   The length of the rectangle must be at least 60 m and no more than 110 m.

Skills and Review 12.1

17. $y = a\sqrt{x}$
   Substitute 1000 for $x$ and 21 for $y$
   $21 = a\sqrt{1000}$
   $21 = 10a$
   $2.1 = a$
   Substitute 2.1 for $a$ and 4.2 for $y$
   $y = 2.1\sqrt{x}$
   $4.2 = 2.1\sqrt{x}$
   $2 = \sqrt{x}$
   $2^2 = (\sqrt{x})^2$
   $8 = x$
19. \( \frac{2}{x} - \frac{3}{x-4} = \frac{5}{2} \)

We wish to combine the terms on the left side of the equation and then solve using the cross multiplication property. Combine the terms on the left side by writing each fraction with a common denominator. The lowest common denominator of \( x \) and \( x-4 \) is \( x(x-4) \).

\[
\begin{align*}
2 \cdot (x-4) &- 3 \cdot x = 5 \\
x \cdot (x-4) &- 2 \\
\frac{2(x-4) - 3x}{x(x-4)} &\cdot \frac{1}{2} \\
\frac{2x - 8 - 3x}{x(x-4)} &\cdot \frac{1}{2} \\
\frac{x - 8}{x^2 - 4x} &\cdot \frac{5}{2} \\
\frac{-x - 8}{x^2 - 4x} &\cdot \frac{5}{2} \\
2(-x - 8) &\cdot \frac{5(x^2 - 4)}{2} \\
-x - 16 &\cdot \frac{5x^2 - 20x}{2x^2 - 18x + 16} \\
0 &\cdot \frac{5x^2 - 18x + 16}{2x - 8} \\
0 &= \frac{5x^2 - 18x + 16}{2x - 8} \\
0 &= 5x^2 - 18x + 16 \\
0 &= 5x^2 - 10x + 6x + 16 \\
0 &= 5x^2 - 8x + 16 \\
5x^2 - 8x + 16 &= 0 \\
5x^2 - 8x &= 10x + 16 \\
5x^2 - 8x + 10x &= 16 \\
5x^2 - 8x + 16 &= 0 \\
x(5x - 8) &= 0 \\
x &= \frac{8}{5} \text{ or } x = 2 \\
2 = 8 &= 5 \text{ or } x = 2 \\
x &= \frac{8}{5} \\
21. \quad 10^{y+3} &= 100 \\
\log(10^{y+3}) &= \log(10^2) \\
y + 3 &= 2 \\
y &= -1 \\
23. \quad Y1 = X^3, \ Y2 = 2 \\
Windows may vary: Zoom Decimal \\
\begin{array}{c}
\text{Intersection} \\
H = 1.259921 \\
V = 2 \\
x = 1.26
\end{array}

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