Section 12.3

1. a, d; these points lie in the shaded region of the graph.

3. Answers may vary; (0, 99) and (20, 60) are solutions because they are in the shaded region of the graph. (0, 100) and (49, 50) are not solutions. (0, 100) is on the boundary of a dashed line and (49, 50) is outside the shaded region.
5. a. ii  b. i  c. iii
Treat the inequalities as equations and graph the lines. Use the inequality symbol to determine if the line is solid or dashed and if the shading is above or below the line.
In (a), the first inequality has a > which indicates shading above a dashed line. The second inequality (\geq) indicates shading above a solid line. Graph ii. fits these criteria.
In (b), the first inequality has a < which indicates shading below a dashed line. The second inequality (\geq) indicates shading above a solid line. Graph i. fits these criteria.
In (c), we have shading above the first solid line (\geq) and shading below the second dashed line (<). Graph iii. fits these criteria.

7. \(y < x - 2\)
Shade below a dashed line for \(y = x - 2\)
\(y \leq 2x + 4\)
Shade below a solid line for \(y = 2x + 4\)
\(y \geq 4\)
Shade above a solid line for \(y = 4\)

9. \(y < 4\) Shade below a dashed line for \(y = 4\)
\(y \geq 4\) Shade above a solid line for \(y = 4\)
\(x < 1\) Shade to the left of a dashed line for \(x = 1\)
\(x \geq 5\) Shade to the right of a solid line for \(x = 5\)

11. Solving both inequalities for \(y\) we have
\(-3y > x + 6\)
\(y < \frac{x}{3} - 2\)
\(x + y \leq 1\)
\(y \leq x + 1\)
This gives the system
\(y < \frac{x}{3} - 2\)
Shade below a dashed line for \(y = \frac{x}{3} - 2\)
\(y \leq x + 1\)
Shade below a solid line for \(y = x + 1\)

13. Solving the inequalities for \(y\) gives
\(y \geq 0\)
Shade above the solid line for \(y = 0\)
\(y < x + 6\)
Shade below the dashed line for \(y = x + 6\)
\(y < 4x - 10\)
Shade below the dashed line for \(y = 4x - 10\)

15. a. Let \(x\) represent the weight of M&Ms (in pounds) and let \(y\) represent the weight of Peppermint Patties (in pounds).
\(x + y \leq 1\) Weight inequality
\(x \geq \frac{1}{4}\) M&Ms are at least .25 pounds
\(y < 2x\) Peppermint Patties are less than twice the M&Ms
\(y \geq 0\) Can't have a negative weight of Peppermint Patties
Solving the inequalities for \(y\) gives
\(y \leq x + 1\)
Shade below a solid line for \(y = x + 1\)
\(x \geq .25\)
Shade to the right of a solid line for \(x = .25\)
\(y < 2x\)
Shade below a dashed line for \(y = 2x\)
\(y \geq 0\)
Shade above a solid line for \(y = 0\)

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17. \( Y_1 = \text{abs}(X - 4), Y_2 = 7 \)
Windows may vary: \( \text{Xmin} = 18.8, \text{Xmax} = 18.8, \text{Ymin} = -10, \text{Ymax} = 10 \).

![Graph of the absolute value function](image)

The graph of the absolute value function \( Y_1 \) is at or below the line \( Y_2 \) when \( X \) is between -3 and 11; \( \{ x \mid -3 \leq x \leq 11 \} \).

19. a. Let \( x \) = the number of hours it takes the first painter to complete the job alone.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{First Painter} & \text{Work} & \text{Time} & \text{Rate} \\
\hline
\text{1 job} & x \text{ hours} & \frac{1}{x} & \text{job/hour} \\
\hline
\text{Second Painter} & \text{1 job} & x + 2 \text{ hours} & \frac{1}{x + 2} & \text{job/hour} \\
\hline
\text{Together} & \text{1 job} & 5 \text{ hours} & \frac{1}{5} & \text{job/hour} \\
\hline
\end{array}
\]

b. \[ \frac{1}{x} + \frac{1}{x + 2} = \frac{1}{5} \]

The sum of the first and second painter's rates equals their rate working together.

c. \[
\frac{1}{x} + \frac{1}{x + 2} = \frac{1}{5} \\
\frac{1 \times (x + 2) + 1 \times x}{x \times (x + 2)} = \frac{1}{5} \\
\frac{x + 2 + x}{x \times (x + 2)} = \frac{1}{5} \\
\frac{2x + 2}{x \times (x + 2)} = \frac{1}{5} \\
\frac{x^2 + 2x}{x \times (x + 2)} = \frac{1}{5} \\
5 \times (2x + 2) = 1 \times (x^2 + 2x) \\
10x + 10 = x^2 + 2x \\
0 = x^2 - 8x - 10
\]

This quadratic doesn't factor with rational coefficients, so we use the quadratic formula. \( a = 1, b = -8, c = -10 \)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
x = \frac{8 \pm \sqrt{(-8)^2 - 4 \times 1 \times -10}}{2 \times 1} \\
x = \frac{8 \pm \sqrt{64 + 40}}{2} \\
x = \frac{8 \pm \sqrt{104}}{2} \\
x = \frac{4 \pm \sqrt{26}}{2} \\
x = 2 \pm \sqrt{13}
\]

\( x \approx 4.5, 4.5 \pm 5.1 \)

\( x = 9.1 \) or \( x = -1.1 \)

d. Ignore the negative solution; it takes the first painter 9 hours to complete the job alone.

21. \[
\frac{x + 4}{x^2 + 8x + 15} = \frac{x^2 - 16}{x + 3} \\
\frac{x + 4}{x^2 + 8x + 15} = \frac{x^2 - 16}{x + 3} \\
\frac{x + 4}{(x + 3)(x + 5)} = \frac{x^2 - 16}{(x - 4)(x + 4)} \\
\frac{x + 4}{(x + 5)(x + 4)} = \frac{1}{(x + 5)(x - 4)}
\]
23. \( y = ax^3 \); \( y \) varies directly as the cube of \( x \).

\[ 9 = a \times 1^3 \quad \text{Substitute 1 for } x \text{ and 9 for } y \]

\[ 9 = a \]

Substituting 9 for \( a \) to write the specific variation equation and substituting 72 for \( y \).

\[ y = 9x^3 \]

\[ 72 = 9x^3 \]

\[ 8 = x^3 \]

\[ 8^{1/3} = (x^3)^{1/3} \]

\[ (2)^{1/3} = (x^3)^{1/3} \]

\[ 2 = x \]

25. \( f(x) = x^2 - 5x - 14 \)

a. \( y \)-intercept = (0, -14); setting \( x = 0 \) gives \( y = -14 \).

b. Set \( y = 0 \) and solve for \( x \)

\[ x^2 - 5x - 14 = 0 \]

\[ x^2 + 2x - 7x - 14 = 0 \]

\[ x(x + 2) - 7(x + 2) = 0 \]

\[ (x + 2)(x - 7) = 0 \]

\[ x + 2 = 0 \text{ or } x - 7 = 0 \]

\[ x = -2 \text{ or } x = 7 \]

\( x \)-intercepts: (-2, 0), (7, 0)

c. You may use the formula \( x = \frac{-b}{2a} \) for the \( x \)-coordinate of the vertex or recall that the \( x \)-coordinate of the vertex is midway between the \( x \)-intercepts. Either method gives

\[ x = \frac{-5}{2} = 2.5 \]. Substituting 2.5 for \( x \) in the original function will give the \( y \)-coordinate of the vertex.

\[ f(2.5) = 2.5^2 - 5 \times 2.5 - 14 = -20.25 \]

\[ \text{Vertex } = \left( \frac{5}{2}, -\frac{81}{4} \right) = (2.5, -20.25) \]

d.