Chapter 5: More Applications of Linear Equations

Section 5.1

1. Two of several proportions: \( \frac{2}{50} = \frac{6}{150} \) or \( \frac{6}{150} = \frac{2}{50} \). Both proportions produce the same cross product.

\[ 2 \times 150 = 6 \times 50 \]
\[ 300 = 300 \]

3. \( \frac{x + 4}{8} = \frac{11}{3} \)

Use the cross multiplication property

\[ 3(x + 4) = 8 \times 11 \]
\[ 3x + 12 = 88 \]
\[ 3x = 76 \]
\[ x = \frac{76}{3} \approx 25.33 \]

5. Because triangle \( ABD \) is similar to \( ECD \), we know that corresponding sides of the triangles have the same ratio. Here is one of several possible proportions that can be set up involving the missing length \( AD \):

\[ \frac{\text{length of } ED}{\text{length of } AD} = \frac{\text{length of } EC}{\text{length of } AB} \]

Let \( x \) represent the length of \( AD \). Substituting the known quantities and \( x \), we have:

\[ 3 \times 7 = 2x \]

Use the cross multiplication property

\[ 21 = 2x \]
\[ x = \frac{21}{2} = 10.5 \]

The length of \( AD \) is 10.5 cm.

7. \( D \) has the same solution as \( A \) because their cross products are the same.

\( A: 6 \times 84 = 7x \)
\( D: 7x = 6 \times 84 \)

By either equation the solutions is \( x = 72 \).

9. Two of several proportions: \( \frac{x}{84} = \frac{6}{7} \) and \( \frac{84}{x} = \frac{7}{6} \)

11. This is a work rate problem similar to Example 4, use the formula,

\[ \text{rate} = \frac{\text{work}}{\text{time}} \]

\[ \text{Laura's rate} = \frac{1}{2} \text{ cleanup} \]
\[ \text{Mom's rate} = \frac{1}{5} \text{ cleanup} \]

Combining their rates we have:

\[ \frac{1}{2} + \frac{1}{5} = \frac{7}{10} \]

This is Laura’s and her mom’s combined work rate, we want the time it takes to finish the cleanup. Let \( t \) = the number of minutes it takes Laura and her mom to pick up the toys together. Using the work rate formula, we have

\[ \text{rate} = \frac{\text{work}}{\text{time}} \]

\[ \frac{7}{10} = \frac{1 \text{ cleanup}}{t} \]

Use the cross multiplication property

\[ 7t = 1 \times 10 \]
\[ 7t = 10 \]
\[ t = \frac{10}{7} \approx 1.4 \]

Working together it takes Laura and her mom about 1.4 minutes to cleanup the toys.

13. Let \( t \) = the number of minutes it takes Latasha to read a 145-page book.

\[ \frac{15}{25} = \frac{145}{t} \]

Use the cross multiplication property

\[ 15t = 25 \times 145 \]
\[ 15t = 3625 \]
\[ t = \frac{725}{3} \approx 242 \]

It takes Latasha about 242 minutes to read the entire 145-page book.
15. Using the similar sampling problem in Example 6, we have the proportion:
\[
\frac{\text{tagged penguins in sample}}{\text{total fish in sample}} = \frac{\text{tagged penguins in region}}{\text{total penguins in region}}
\]
Let \( p \) = the population of penguins in the region.

\[
\frac{4}{17} = \frac{20}{p}
\]
\( 4p = 17 \times 20 \) Use the cross multiplication property
\( 4p = 340 \) Solve for \( p \)
\( p = 85 \)
We estimate the penguin population of the region to be 85.

**Skills and Review 5.1**

17. Let \( x \) = the measure of the smaller angle, and \( y \) = the measure of the larger angle.

\[
x + y = 180
\]
\( y = 4x + 10 \)

We chose substitution to solve this system, because an equation is in slope-intercept form.
Substituting \( 4x + 10 \) for \( y \) into \( x + y = 180 \)
\( x + 4x + 10 = 180 \) Solve for \( x \)
\( 5x = 170 \)
\( x = 34 \)
Substitute 34 for \( x \) into \( y = 4x + 10 \)
\( y = 4 \times 34 + 10 \)
\( y = 136 + 10 \)
\( y = 146 \)
The smaller angle measures 34°, and the larger angle measures 146°.

19. a. Use the slope formula to find the slope given the two points: \((x_1, y_1) = (2, 3)\), \((x_2, y_2) = (5, 1)\).

\[
m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{5 - 2} = \frac{-2}{3}
\]

b. Substitute the value for the slope and the values of \( x \) and \( y \) from either point into the slope intercept formula. Then solve for \( b \).
\[
y = mx + b \quad \text{when} \quad m = \frac{4}{3}, \quad x = 5 \quad \text{and} \quad y = 1
\]
\[
1 = \frac{4}{3} \times 5 + b
\]
\[
1 = \frac{20}{3} + b
\]
\[
1 - \frac{20}{3} = b
\]
\[
\frac{3}{3} - \frac{20}{3} = b
\]
\[
-\frac{17}{3} = b
\]
Write an equation by substituting the given value for the slope and the found value of \( b \) into the slope-intercept formula.
\[
y = mx + b \quad \text{when} \quad m = \frac{4}{3} \quad \text{and} \quad b = -\frac{17}{3}
\]
\[
y = \frac{4}{3} \times \frac{-17}{3}
\]

21. \[
\frac{2x - 5}{3} = 5x
\]
\[
\frac{2x - 5}{3} = \frac{5x}{1}
\]
Write 5x as the fraction \( \frac{5x}{1} \)
\[
1 \times (2x - 5) = 5 \times 5x
\]
Use the cross multiplication property
\( 2x - 5 = 15x \) Solve for \( x \)
\[
-13x = 5
\]
\[
x = \frac{-5}{13}
\]
\[
-0.38 \approx x
\]

23. Because \( \pi \) is a nonrepeating, nonterminating decimal, the only way to find an exact area of this circle is to leave the \( \pi \) symbol in our simplified answer.
\[
A = \pi r^2
\]
\[
= \pi (6)^2
\]
\[
= \pi 36
\]
\[
= 36\pi \text{ inches}^2
\]

25. From Section 1.4, a formula for the area of a parallelogram is
\[
A = \text{base} \times \text{height}
\]
when base = 11 mm and height = 7 mm

\[
A = 11 \times 7
\]
\[
A = 77 \text{ mm}^2
\]