5. a. \( x^2 + 6 \) = \( 3x + 2x \)
6x^2 = 6x^2; the diagonal products are equal, so the four terms factor.

b.

\[
\begin{array}{c|c}
\hline 
\hline 
-3 & 2x \\
\hline 
-6 & x \hline 
\end{array}
\]

\( x^2 - 5x + 6 = (x - 3)(x - 2) \)

c. George made his mistake in the grouping of the second step -2x + 6
Use the rule for subtraction to write the four terms as a sum:

\[
x^2 - 5x + 6 = x^2 - 3x - 2x + 6 = (x - 3)(x - 2)
\]

7. 6x^2 - 11x - 10; arranging the terms in descending order.

Diagonal product = 6x^2 \((-10) = 60x^2. Look for a factor pair of 60x^2 that adds to the middle term, -11x. The diagonal product is negative so we need one positive and one negative factor. The middle term is negative so the negative factor must have a larger absolute value than the positive factor.

\[
\begin{array}{c|c}
1*60 & 1x + 6x = 7x \text{ Yes} \\
2*30 & 2x + 3x = 5x \text{ No} \\
3*20 & 3x + 20x = 23x \text{ No} \\
4*15 & 4x + 15x = 19x \text{ Yes} \\
5*12 & 5x + 12x = 17x \text{ No} \\
6*10 & 6x + 10x = 16x \text{ No} \\
\end{array}
\]

6x^2 - 11x - 10 = 6x^2 + 4x + (-15x + -10) Split the middle term

\[
\begin{array}{c|c}
\hline 
\hline 
3x & 2x \\
\hline 
-5 & 6x \hline 
\end{array}
\]

6x^2 - 11x + 10 = (3x + 2)(2x - 5)

Or, factor symbolically by grouping.

\[
6x^2 - 11x - 10 = 6x^2 + 4x + (-15x + -10) \text{ Split middle term}
\]

\[
(6x^2 + 4x) + (-15x + -10) \text{ Form groups of two terms}
\]

\[
2x(3x + 2) + (-5)(3x + 2) \text{ Factor GCF from each group}
\]

\[
=(3x + 2)(2x - 5) \text{ Factor out common binomial, (3x + 2)}
\]
9. Diagonal product \( = 5x^2 \times 8 = 40x^2 \). Look for a factor pair of \( 40x^2 \) that adds to the middle term, \( 3x \). The diagonal product is positive and the middle term is negative so both factors need to be negative.

\[
egin{array}{l}
1 \times 40 = 40 \\
2 \times 20 = 40 \\
4 \times 10 = 40 \\
5 \times 8 = 40 \\
\end{array}
\]

\(1x + 40x = 41x\) No
\(2x + 20x = 22x\) No
\(4x + 10x = 14x\) No
\(5x + 8x = 13x\) No
There are no factor pairs of \( 40x^2 \) that add to \( 3x \), so \( 5x^2 - 3x + 8 \) cannot be factored with integers.
This is a prime polynomial.

11. Diagonal product \( = 2x^6 \times \frac{28}{x^6} = 56 \). Look for a factor pair of \( 56 \) that adds to the middle term, \( 1x \). The diagonal product is negative so we need one positive factor and one negative factor. The middle term is negative to the negative factor must have a larger absolute value than the positive factor.

\[
egin{array}{l}
1 \times 56 = 56 \\
2 \times 28 = 56 \\
4 \times 14 = 56 \\
7 \times  \frac{8}{x^6} = 56 \\
\end{array}
\]

\(1x + 56 = -56\) No
\(2x + 28 = -56\) No
\(4x + 14x = -10\) No
\(7x + \frac{8}{x^6} = 1x\) Yes
\(2x^6 - x^2 - 28 = 2x^6 + 7x^2 + 8x^2 + 28\) Split the middle term
\[
\begin{array}{|c|c|c|}
\hline
x^2 & 2x^6 & 7x^2 \\
\hline
4 & -8x^2 & -28 \\
\hline
\end{array}
\]
\(2x^2 - x^2 - 28 = (2x^2 + 7)(x^2 - 4)\)
Or, factor symbolically by grouping.
\(2x^2 - x^2 - 28\)
\(= 2x^2 + 7x^2 + 8x^2 - 28\) Split middle term
\((2x^2 + 7x^2) + (8x^2 - 28)\) Form groups of two terms
\(= x^2(2x^2 + 7) + 4(2x^2 + 7)\) Factor GCF from each group
\(= (2x^2 + 7)(x^2 - 4)\) Factor out common binomial, \((2x^2 + 7)\)

13. a. Factor the left side of the equation. The diagonal product is \( 32x^2 \). Look for a factor pair of \( 32x^2 \) that adds to \( -12x \). The diagonal product is positive and the middle term is negative so both factors must be negative.

There are four pairs of factors; only \(-4x\) and \(8x\) add to \(-12x\).
\[
\begin{array}{l}
\left(x^2 - 4x - 8x + 32\right) = 0 \\
\left(x^3 + 4x\right) + \left(8x + 32\right) = 0 \\
x(x - 4) - 8(x - 4) = 0 \\
(x - 4)(x - 8) = 0 \\
\end{array}
\]
\(x - 4 = 0\) or \(x - 8 = 0\)
\(x = 4\) or \(x = 8\)

b. Arranging the terms in descending order on one side of the equation, we have \( x^2 - 3x - 10 = 0 \)
\(2x\) and \(5x\) is the factor pair of \(10x^2 \) that adds to \(3x\).
\[
\begin{array}{l}
\left(x^2 + 2x\right) + \left(5x + 10\right) = 0 \\
\left(x^2 + 2x\right) + \left(5x + 10\right) = 0 \\
x(x + 2) - 5(x + 2) = 0 \\
(x + 2)(x - 5) = 0 \\
\end{array}
\]
\(x + 2 = 0\) or \(x - 5 = 0\)
\(x = -2\) or \(x = 5\)

c. \(1x\) and \(6x\) is the factor pair of \(6x^2\) that adds to \(7x\).
\[
\begin{array}{l}
\left(2x^2 + 7x + 3\right) = 0 \\
\left(2x^2 + 7x + 3\right) + \left(6x + 3\right) = 0 \\
x(2x + 1) + 3(2x + 1) = 0 \\
(2x + 1)(x + 3) = 0 \\
\end{array}
\]
\(2x + 1 = 0\) or \(x + 3 = 0\)
\(2x = -1\) or \(x = -3\)
\(x = \frac{-1}{2}\) or \(x = -3\)
15. a. 
\[ x^2 + x - 2 = (x + 2)(x - 1) \]
\[ (-2)^2 + x - 2 = (-2 + 2)(-2 - 1) \]
\[ 4 - 2 - 2 = 0 \times 3 \]
\[ 0 = 0 \text{ OK!} \]

The expressions appear to be equivalent because they have the same output for various values of \( x \).

b. x-axis
\[ (-2, 0) \quad (1, 0) \]

e. \( x = 6 \) or \( x = -3 \)

d. It’s easy to recognize the values of \( x \) that give an output of 0 in a factored polynomial.

Skills and Review 6.4

17. \[
\begin{array}{c|c|c|c}
\text{x} & \text{3} \\
\hline
x^2 & 9 \\
\hline
3x & 9 \\
\hline
(x + 3)^2 = x^2 + 6x + 9
\end{array}
\]

19. \[
7x^2 - 21x = 0 \\
7x(x - 3) = 0 \\
x = 0 \text{ or } x = 3
\]

21. We are looking for the time of day when Linda and Lucy meet, but we must first find, how long they travel.

\[
\begin{array}{c|c|c}
\text{Linda} & \text{1/8} \text{ miles} & \text{1/4} \text{ miles} \\
\hline
\text{Lucy} & \text{1/8} \text{ miles} & \text{1/4} \text{ miles} \\
\hline
\text{Total Distance Traveled} & \text{3/4 mile} & \text{3/4 mile} \\
\hline
\text{5:00 P.M.} & \text{5:08 P.M.} & \text{5:05 P.M.}
\end{array}
\]

Let \( t \) = the number of minutes that Linda runs. Lucy starts 3 minutes later, so her running time is represented by \( t - 3 \). This is a distance, rate, and time problem. When Linda and Lucy meet their total distance traveled is \( \frac{3}{4} \) mile. Linda travels a distance of \( \frac{1}{8} \) miles and Lucy travels a distance of \( \frac{1}{4} \) (\( t - 3 \)) miles.

Summing their distances gives the equation:
\[
\frac{1}{8} t + \frac{1}{4} (t - 3) = \frac{3}{4}
\]
\[
\frac{1}{8} t + \frac{1}{4} t - \frac{3}{4} = \frac{3}{4}
\]
\[
\frac{2}{8} t + \frac{2}{4} t = \frac{3}{4}
\]
\[
\frac{2}{8} t = \frac{3}{8}
\]
\[
\frac{3}{8} t = \frac{3}{2}
\]
\[
\frac{8}{3} t = \frac{8}{3}
\]
\[
\frac{3}{8} t = \frac{3}{2}
\]
\[
t = 4
\]

Linda runs for 4 minutes. She started running at 5 P.M., so they meet at 5:04 P.M.
23. \( x + \frac{1}{2} y = 6 \)

\[ y = -2x + 10 \]

Method 1: Place both equations in slope-intercept form and compare slopes.

\[ y = -2x + 12 \]
\[ y = -2x + 10 \]

The equations have the same slope with different y-intercepts so the lines are parallel and the system has no solution.

Method 2: Solve the system algebraically.

Substitute the second equation into the first equation.

\[ x + \frac{1}{2}(-2x + 10) = 6 \]
\[ x - x + 5 = 0 \]
\[ 5 = 6 \]

But 5 ≠ 6 so there is no solution to this system of equations.

25. a. \( 11 - (x + 2) = 5(x - 7) \)

\[ 11 - x - 2 = 5x - 35 \]
\[ 9 - x = 5x - 35 \]
\[ 44 - x = 5x \]
\[ 44 = 6x \]
\[ \frac{44}{6} = x \]
\[ \frac{22}{3} = x \]

b. \( x = 7.333 \)