Chapter 7: Quadratic Equations and Graphs

Section 7.1

1. a. See Figure 2 on page 259 of the text. Note that some data are rounded.
   
   b. No
   
   c. No, linear models have constant slope. The graph of these data has changing slope.

3. Let \( s \) be the length of a side of the square.

\[
A = s^2
\]

\[
64 \text{ inches}^2 = s^2
\]

\[
\pm \sqrt{64} \text{ inches} = \sqrt{s^2}
\]

\[
\pm 8 \text{ inches} = s
\]

Ignore the negative solution; a side of the square is 8 inches.

5. a. \( 9 = x^2 \)

\[
\pm \sqrt{9} = \sqrt{x^2}
\]

\[
\pm 3 = x
\]

b. \( 25 = s^2 \)

\[
\pm \sqrt{25} = \sqrt{s^2}
\]

\[
\pm 5 = s
\]

c. \( 26 = s^2 \)

\[
\pm \sqrt{26} = \sqrt{s^2}
\]

\[
\pm 5.1 = s
\]

The solutions for (a) and (b) are exact, but the solutions for (c) are approximate; because 9 and 25 are perfect squares, but 26 is not.

7. \( 400 = 1.2d^2 \)

\[
\frac{400}{1.2} = d^2
\]

\[
\frac{1000}{3} = d^2 \quad \text{Use Math Frac feature}
\]

\[
\pm \frac{1000}{3} = d \quad \text{This is exact}
\]

\[
\pm 18.26 = d
\]

This is approximate because \( \frac{1000}{3} \) is not a perfect square.

9. After entering the expressions in Y1 and Y2 and setting the window variables, we can TRACE the graph or use 2nd CALC intersect to find the approximate solutions.

The points of intersection are approximately (4.08, 5) and (4.08, 5), so the solutions to the equation \( 5 \cdot x^2 = 0 \) are \( x \approx \pm 4.08 \).

11. Linear \hspace{1cm} Quadratic

\[
2x - 3 = 0 \quad 2x^2 - 3 = 0
\]

\[
2x = 3 \quad 2x^2 = 3
\]

\[
x = \frac{3}{2} = 1.5 \quad x^2 = \frac{3}{2}
\]

\[
x = \pm \frac{3}{2}
\]

Solving quadratic equations of the form \( ax^2 + c = 0 \) uses the same process as solving linear equations, with the extra step of taking a square root on both sides of the equation.
13. Each of these quadratic equations can be solved by factoring, see page 240 of the text for review.

a. \[ x^2 + 4x + 3 = 0 \]
Diagonal product = 3x²
1x, 3x is the factor pair of 3x² that adds to 4x (the middle term).
\[
\begin{align*}
x^2 + 4x + 3 & = 0 \\
x^2 + 1x + 3x + 3 & = 0 \\
(x^2 + 1x) + (3x + 3) & = 0 \\
x(x + 1) + 3(x + 1) & = 0 \\
(x + 1)(x + 3) & = 0 \\
x + 1 = 0 \quad \text{or} \quad x + 3 = 0 \\
x = -1 \quad \text{or} \quad x = -3
\end{align*}
\]

b. \[ x^2 - 3x + 2 = 0 \]
Diagonal product = 2x²
1x, -2x is the factor pair of 2x² that adds to -3x.
\[
\begin{align*}
x^2 - 3x + 2 & = 0 \\
x^2 - 1x - 2x + 2 & = 0 \\
(x^2 - 1x) + (-2x + 2) & = 0 \\
x(x - 1) + -2(x - 1) & = 0 \\
(x - 1)(x - 2) & = 0 \\
x - 1 = 0 \quad \text{or} \quad x - 2 = 0 \\
x = 1 \quad \text{or} \quad x = 2
\end{align*}
\]

c. \[ y^2 + 5y - 14 = 0 \]
Diagonal product = -14y²
2y, -7y is the factor pair of -14y² that adds to 5y.
\[
\begin{align*}
y^2 + 5y - 14 & = 0 \\
y^2 - 2y + 7y - 14 & = 0 \\
(y^2 - 2y) + (7y - 14) & = 0 \\
y(y - 2) + 7(y - 2) & = 0 \\
(y - 2)(y + 7) & = 0 \\
y - 2 = 0 \quad \text{or} \quad y + 7 = 0 \\
y = 2 \quad \text{or} \quad y = -7
\end{align*}
\]

d. \[ x^2 - 11x = -18 \]
\[ x^2 - 11x + 18 = 0 \]
Diagonal product = 18x²
-2x, 9x is the factor pair of 18x² that adds to -11x.
\[
\begin{align*}
x^2 - 11x + 18 & = 0 \\
x^2 - 2x - 9x + 18 & = 0 \\
(x^2 - 2x) + (-9x + 18) & = 0 \\
x(x - 2) + -9(x - 2) & = 0 \\
x - 2 = 0 \quad \text{or} \quad x - 9 = 0 \\
x = 2 \quad \text{or} \quad x = 9
\end{align*}
\]

15. When a quadratic equation has only an \( x^2 \)-term and a constant, we can solve the equation by isolating the \( x^2 \)-term and taking the square root on both sides.

a. \[ 4x^2 - 49 = 0 \]
\[ 4x^2 = 49 \]
\[ \sqrt{4x^2} = \pm\sqrt{49} \]
\[ 2x = \pm 7 \]
\[ x = \pm \frac{7}{2} \]

b. Using the difference of two squares, see p 244 of the text for review
\[ 4x^2 - 49 = 0 \]
\[ (2x - 7)(2x + 7) = 0 \]
\[ 2x - 7 = 0 \quad \text{or} \quad 2x + 7 = 0 \]
\[ 2x = 7 \quad \text{or} \quad x = -\frac{7}{2} \]
\[ x = \frac{7}{2} \quad \text{or} \quad x = -\frac{7}{2} \]

c. The solutions from parts (b) and (c) are the same.
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17. We can solve this quadratic using either the square root principle or the difference of two squares.

Square Root Principle

\[ x^2 - 64 = 0 \]
\[ x^2 = 64 \]
\[ x = \pm \sqrt{64} \]
\[ x = \pm 8 \]

Difference of Two Squares

\[ x^2 - 64 = 0 \]
\[ (x - 8)(x + 8) = 0 \]
\[ x - 8 = 0 \text{ or } x + 8 = 0 \]
\[ x = 8 \text{ or } x = -8 \]

19. \[(2x - 3)^2 = (2x - 3)(2x - 3)\]
\[= 4x^2 - 6x - 6x + 9 \]
\[= 4x^2 - 12x + 9 \]

21. a. \[x + y = 212\]

b. \[x - y = 83\]

c. Solve each equation for \( y \) to enter into the \( Y= \) menu.

\[ Y1 = X + 212, Y2 = X - 83 \]

Windows may vary: \( X_{\text{min}} = -250, X_{\text{max}} = 250, Y_{\text{min}} = -250, Y_{\text{max}} = 250 \)

Graphical solutions are approximate: \( x = 147.5 \) and \( y = 64.5 \).

The first number is 147.5 and the second number is 64.5.

23. \[ z - \frac{3}{4} = \frac{1}{3} + \frac{1}{2} \]
\[ \frac{2}{5} \]
\[ \frac{z - 3}{5} = \frac{1}{2} \]
\[ \frac{2}{3} \]
\[ \frac{z = 5}{4} \]
\[ \frac{8}{8} \]
\[ z = 1.875 \]

Check:

Substituting \( \frac{15}{8} \) for \( z \) into the original equation

\[ \frac{15}{8} - \frac{3}{4} = \frac{15}{3} + \frac{1}{2} \]
\[ \frac{15}{8} - \frac{5}{4} = \frac{5}{8} + \frac{4}{8} \]
\[ \frac{9}{8} = \frac{9}{8} \]

25. \[ \frac{(-7) + \sqrt{(-7)^2 - 4 \times 1 \times 12}}{2 \times 1} \]
\[ = \frac{7 + \sqrt{49 - 48}}{2} \]
\[ = \frac{7}{2} \]
\[ = \frac{7}{2} \]
\[ = \frac{7}{2} \]
\[ = \frac{7}{2} \]
\[ = \frac{7}{2} \]
\[ = \frac{7}{2} \]
\[ = \frac{7}{2} \]
\[ = \frac{7}{2} \]