Chapter 9: Powers and Roots

Section 9.1

1. \[16^{-1} = \frac{1}{16}\]
   \[16^0 = 1\]
   \[16^1 = 16\]
   a. \(1 < 16^{1/2} < 16\)
   b. \(\sqrt{16}\)
   c. -4 does not lie between 1 and 16, see part (a).
   d. No, the solutions of \(x^2 = 16\) are \(x = 4\) and \(x = -4\), whereas the solution of \(x = \sqrt{16}\) is only \(x = 4\).

3. a. \(\sqrt[3]{7/2} = \left(\frac{7^{1/3}}{2}\right)^{1/2} = \frac{\sqrt{7}}{2} \quad \text{or} \quad \left(\frac{\sqrt[3]{7}}{2}\right)^{1/2}
   \quad \text{or} \quad \frac{\sqrt{7}}{2}\)
   b. \(\sqrt[2]{3}\)
   c. \(5^{1/2}\)
   d. \(11^{1/5}\)

5. \(\sqrt{(-5)^2} = 5\), but \(\left(\sqrt{-5}\right)^2\) is undefined.
   The two expressions differ in the placement of the parentheses. In the first, we square -5 before taking the square root. In the second, we must attempt the square root of -5 before squaring. Negative numbers, however, do not have real number square roots.
   \(\sqrt{(-5)^2} = -5\) and \(\left(\sqrt{-5}\right)^2\) = -5
   Both these expressions result in taking a cube root of a negative number. Because the cube of a negative number is negative, the cube root of a negative number is a negative number. Section 9.2 provides further information on even and odd roots.

7. a. 16 = 2*2*2*2; therefore 16\(^{1/4}\) = 2.
   b. 216 = 2*2*2*3*3*3; therefore 216\(^{1/3}\) = 2*3 = 6
   c. 64 = 2*2*2*2*2*2; therefore 64\(^{1/6}\) = 2*2 = 4
   d. 32 = 2*2*2*2*2; therefore 32\(^{1/5}\) = 2

9. Each side of the cube is exactly \(\sqrt[3]{20}\) cm or approximately 2.7 cm.

11. a. \(27^{2/3} = (27^{1/3})^2 = 3^2 = 9\)
    b. \(16^{3/4} = (16^{1/4})^3 = 2^3 = 8\)
    c. .00032
    d. \(\left(\frac{9}{25}\right)^{3/2} = \left(\frac{9^{3/2}}{25^{3/2}}\right) = \left(\frac{27}{125}\right) = \frac{3^3}{5^3} = \frac{1.43}{125}\)
    e. 129.64

13. a. \(\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}\)
    b. \(\sqrt[3]{2} = \sqrt[3]{2} \cdot \sqrt[3]{2} = \sqrt[3]{2^2} = \frac{\sqrt{2}}{2}\)
    c. \(\sqrt[3]{\frac{2}{3}} = \frac{\sqrt[3]{2} \cdot \sqrt[3]{3}}{\sqrt[3]{3}} = \frac{2}{\sqrt[3]{3}} = \frac{2\sqrt[3]{3}}{3}\)
    d. \(\sqrt[6]{\frac{5}{6}} = \sqrt[6]{\frac{5 \cdot \sqrt{6}}{6 \cdot \sqrt{6}}} = \sqrt[6]{\frac{5}{6} \cdot \sqrt{6}} = \frac{\sqrt{2\sqrt{6}}}{2}\)
    e. \(\sqrt[6]{\frac{5}{6} \cdot \sqrt{6} \cdot \sqrt{7}} = \frac{\sqrt{2\sqrt{7}}} {2}\)
    f. \(\sqrt[2]{\frac{5}{6} \cdot \sqrt{6} \cdot \sqrt{7} \cdot \sqrt{7}} = \frac{\sqrt{2\sqrt{7}}} {2}\)

15. a. \(7^{1/2} = (7^{1/2})^3 = (7^{1/3} \cdot 7^{1/3} \cdot 7^{1/3}) = 7 \cdot 7 = 49\)
    b. \(7^{1/2} = 49\sqrt{7} = 49 \cdot 2.6458 = 129.64\)

Skills and Review 9.1

17. a. 0
    b. 3 is not in the domain of \(g(x)\);
    \(g(3) = \sqrt{3 - 4} = \sqrt{-1}\), and negative numbers do not have real square roots.

19. \[
   \begin{array}{c}
   \text{Factor the left side} \\
   \text{Apply the Zero Product Property} \\
   \text{An alternative is to use the quadratic formula.}
   \end{array}
   \]

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23. a. \(x^2 - 10x + 24\)
   Factor pair of the diagonal product, \(24x^2\), that sums to \(-10x\) is \(-4x\) and \(-6x\):
   \[= x^2 - 4x - 6x + 24\]
   Split the middle term
   \[= (x^2 - 4x) + (-6x + 24)\]
   Form groups of two terms
   \[= x(x - 4) - 6(x - 4)\]
   Factor the GCF from each group
   \[= (x - 4)(x - 6)\]
   Factor out the common binomial, \((x - 4)\)

b. \(-4x^2 + 36\)
   \[= 4(x^2 - 9)\]
   Factor the GCF from both terms
   \[= 4(x - 3)(x + 3)\]
   Use the difference of two squares

c. \(10x^2 - 35x\)
   \[= 5x(2x - 7)\]
   Factor the GCF from both terms

25. \(6x - 7(1 + x) = 5 - 3x\)
   \[6x - 7 - 7x = 5 - 3x\]
   Use the distributive property, LS
   \[-x - 7 = 5 - 3x\]
   Combine like terms, LS
   \[2x - 7 = 5\]
   Add 3x on both sides
   \[2x = 12\]
   Add 7 on both sides
   \[x = 6\]
   Divide by 2 on both sides