Review Chapter 5

1. a. \[
\frac{10}{4} = \frac{x}{18}
\]
   \[10 \times 18 = 4x\]
   \[180 = 4x\]
   \[45 = x\]

b. \[
\frac{9}{x} = \frac{2}{3}
\]
   \[9 \times 3 = 2x\]
   \[27 = 2x\]
   \[x = \frac{27}{2} = 13.5\]

c. \[
\frac{x}{5} = 3
\]
   \[x = 3 \times 5\]
   \[x = 15\]

d. \[
\frac{5}{x+1} = \frac{8}{2}
\]
   \[5(x + 1) = 2 \times 8\]
   \[5x + 5 = 16\]
   \[5x = 11\]
   \[x = \frac{11}{5} = 2.2\]
2. Two of several possible proportions:
\[
\frac{6 \text{ gallons}}{9 \text{ dollars}} = \frac{20 \text{ gallons}}{c \text{ dollars}} \quad \text{or} \quad \frac{9 \text{ dollars}}{6 \text{ gallons}} = \frac{c \text{ dollars}}{20 \text{ gallons}}
\]

3. \( C = \frac{F - 32}{1.8} \) when \( C = 20^\circ \)
\[
20 = \frac{F - 32}{1.8} \\
20 = \frac{F - 32}{1.8} \\
1 = \frac{F - 32}{1.8} \\
20 \times 1.8 = F - 32 \\
36 = F - 32 \\
68 = F
\]
68°F is equivalent to 20°C.

4. Cost of 1 gallon of heating oil is
\[
\frac{\$210}{175 \text{ gallons}} = \$1.2 \text{ per gallon}
\]
Cost of 150 gallons of heating oil is 1.2 \times 150 or $180.

5. \( \frac{\text{height of person}}{\text{length of person's shadow}} = \frac{\text{height of building}}{\text{length of building shadow}} \)
Let \( h = \) the height of the building (in feet).
\[
\frac{5}{4} = h \\
5 \times 4 = 4h \\
21.25 = h
\]
The building is 21.25 feet tall.

6. Let \( c = \) the number of gallons of gasoline that can be bought for $18.
\[
\frac{25}{15} = \frac{18}{c} \\
25c = 15 \times 18 \\
25c = 270 \\
c = 10.8
\]
10.8 gallons of gasoline can be bought for $18.

7. a. The time it takes for two people to complete a job is less than the fastest person working alone.

b. First find Sid's and Joanne's work rate, then find their combined rate. Then set up and solve a proportion to find the time it takes for Sid and Joanne to complete the job together.

\[
\text{rate} = \frac{\text{work}}{\text{time}} \\
\text{Sid's rate} = \frac{1 \text{ job}}{2 \text{ hours}} \\
\text{Joanne's rate} = \frac{1 \text{ job}}{6 \text{ hours}}
\]
Combined rate = \( \frac{1}{2} + \frac{1}{6} = \frac{2}{3} \)
Let \( t = \) the number of hours it takes Sid and Joanne to complete the job together.
\[
\frac{2}{3} = \frac{1 \text{ job}}{t} \\
2t = 3 \times 1 \\
2t = 3 \\
t = \frac{3}{2} = 1.5
\]
Working together Sid and Joanne can complete 1 job in \( \frac{3}{2} \) hours.

8. \( \frac{\text{tagged deer in sample}}{\text{total deer in sample}} = \frac{\text{tagged deer in forest}}{\text{total deer in forest}} \)
Let \( p = \) the population of deer in the forest.
\[
\frac{9}{22} = \frac{42}{p} \\
9p = 22 \times 42 \\
9p = 924 \\
p = 102.7
\]
We estimate the deer population to be 103.

9. a. \( \text{part} = \text{percent} \times \text{whole} \)

b. Dividing the formula from (a) by percent on both sides, we have: \( \text{whole} = \frac{\text{part}}{\text{percent}} \)
10. 

11. We estimate 50% because 14 is a little less than 15 which is 50% of 30.

12. \[ \text{percent} = \frac{\text{part}}{\text{whole}} \]
\[ = \frac{14}{30} = \frac{46}{3} \approx 46.67\% \]
14 is \( \frac{46}{3} \% \) of 30.

13. a. 

\[ \text{part} = \text{percent} \times \text{whole} \]
\[ = 0.15 \times 20 = 3 \]
One serving of peanut butter has 3 g of saturated fat.

b. We estimate 25% because 21 is a little more than 20 which is 25% of 80.

c. \[ \text{percent} = \frac{\text{part}}{\text{whole}} \]
\[ = \frac{21}{80} = 0.2625 = 26.25\% \]
21 is 26.25% of 80.

14. 

15. \[ \text{part} = \text{percent} \times \text{whole} \]
\[ = 0.06 \times 200 = 12 \]
12 is 6% of 200.

16. 

\[ \text{percent} = \frac{\text{part}}{\text{whole}} \]
\[ = \frac{18}{200} = 0.09 = 9\% \]
The strength of the solution is 9%.

18. 

2000 = 0.08 \times \text{whole}
2000 = \text{whole}
25000 = \text{whole}
The RDA for fiber is 25000 mg.

19. a. The solute is the substance that is dissolved in another.

b. The solvent is the substance in which another is dissolved.

c. The solution consists of the solute and the solvent mixed together.

20. Strength of a solution is the ratio of the mass of solute to the mass of solution expressed as a percent.

\[ \text{strength} = \frac{\text{mass solute}}{\text{mass solution}} \]
\[ \text{percent} = \frac{\text{part}}{\text{whole}} \]
\[ = \frac{18}{200} = 0.09 = 9\% \]
The strength of the solution is 9%.
21. Taxes and surcharges of 12.5% mean the total phone bill was 12.5% of the original price (charge for phone calls).

Taxes and surcharges of 12.5% mean the total phone bill was 12.5% of the original price (charge for phone calls).

\[
\text{total phone bill} = 1.125 \times \$29.65
\]

\[
\text{total phone bill} \approx \$33.36
\]

Rounded to the nearest cent, the total cost is \$33.36.

22. The sale price is 72.62% of the original price.

\[
\text{sale price} = \frac{305}{420} = 0.7262 = 72.62\%
\]

The sale price is 72.62% of the original price. Thus the discount percent is 100% - 72.62% or 27.38%.

23. Because the population increased 17%, the new population is 117% of the original population.

\[
\text{new population} = 1.17 \times \text{original population}
\]

\[
\frac{1.17 \times \text{original population}}{\text{original population}} = 12500
\]

\[
\text{original population} = \frac{12500}{1.17} \approx 10684
\]

The original population was approximately 10684 people.

24. a. Let \( x \) = the amount of peanut butter fudge (in pounds) and let \( y \) = the amount of vanilla fudge (in pounds).

\[
x + y = 2 \quad \text{Weight equation}
\]

\[
5x + 7y = 13 \quad \text{Money equation}
\]

b. The customer can purchase .5 pounds of peanut butter fudge and 1.5 pounds of vanilla fudge.

c. \( Y_1 = X + 2 \)

\[
Y_2 = \left(\frac{5}{7}\right)X + 13/7
\]

d. Windows may vary: Zoom Decimal

25. Let \( x \) = the number of mL of 10% solution and let \( y \) = the number of mL of 30% solution.

\[
x + y = 200 \quad \text{Solution equation}
\]

\[
.10x + .30y = .22 \times 200 \quad \text{Acid equation}
\]

We chose to solve the system by substitution. Solving the first equation for \( y \) in terms of \( x \)

\[
y = 200 - x
\]

Substituting 200 - \( x \) for \( y \) into \( .10x + .30y = 44 \)

\[
.10x + .30(200 - x) = 44
\]

\[
.10x + 60 - .30x = 44
\]

\[
-.20x + 60 = 44
\]

\[
-.20x = 16
\]

\[
x = 80
\]

\[
y = 200 - 80
\]

We need 80 mL of the 10% solution and 120 mL of the 30% solution.
26. Let $x$ = the amount of money (in dollars) in the checking account and let $y$ = the amount of money (in dollars) in the savings account.

\[ x + y = 700 \]  \hspace{1cm} \text{Investment equation}

\[ .023x + .054y = 34.30 \]  \hspace{1cm} \text{Interest equation}

We chose to solve the system by elimination. Multiply the first equation by $- .023$ on both sides and add. The $x$-term in the resulting equation will be eliminated.

\[ -.023(x + y) = -.023(700) \]

\[ .023x + .054y = 34.30 \]

\[ -.023x - .023y = -7.5 \]

\[ .031y = 18.2 \]

\[ y = 587.10 \]

\[ x + 587.10 = 700 \]  \hspace{1cm} \text{Substitute 587.10 for $y$ into $x + y = 700$}

\[ x = 112.90 \]

Rounded to the nearest cent, $112.90 was deposited in the checking account and $587.10 was deposited in the savings account.

27. \[ 3y + z = 5 \]

\[ -x + 2y + 4z = -6 \]

\[ 2x + 5y - 3z = 8 \]

Writing the equations in standard form, we have

\[ 0x + 3y + 1z = 5 \]

\[ -1x + 2y + 4z = -6 \]

\[ 2x + 5y - 3z = 8 \]

Solving with the MATRIX feature on the TI-83, we have

\[ x = 22.33, y = 4.83, z = -9.5 \]

28. Let $x$ = the number of pennies, let $y$ = the number of nickels, and let $z$ = the number of quarters.

\[ x + y + z = 31 \]  \hspace{1cm} \text{Coins equation}

\[ .01x + .05y + .25z = 1.51 \]  \hspace{1cm} \text{Money equation}

\[ y = 4x \]  \hspace{1cm} \text{Nickels and pennies equation}

The equations in standard form:

\[ 1x + 1y + 1z = 31 \]

\[ .01x + .05y + .25z = 1.51 \]

\[ 4x + 1y + 0z = 0 \]

Solving with the MATRIX feature on the TI-83 calculator, we have

\[ x = 6, y = 24, \text{ and } z = 1 \]

Giovanni has 6 pennies, 24 nickels, and 1 quarter.

Chapter 5 Test

1. \[ \frac{10}{x} = \frac{3}{8} \]

\[ 10 \times 8 = 3x \]

\[ x = \frac{80}{3} = 26.67 \]

2. Let $x$ = the number of U.S. dollars that can be purchased for 200 euros.

\[ \frac{50}{42.9} = \frac{x}{200} \]

\[ 50 \times 200 = 42.9x \]

\[ 10000 = 42.9x \]

\[ x = 10000 = 233.10 \]

We can purchase about 233.10 U.S. dollars for 200 euros.

3. \[ \text{rate} = \frac{\text{work}}{\text{time}} \]

Ladeen's rate = \[ \frac{1 \text{ route}}{6 \text{ hours}} \]

Larry's rate = \[ \frac{1 \text{ route}}{8 \text{ hours}} \]

Combined rate = \[ \frac{1}{6} + \frac{1}{8} = \frac{7}{24} \]

Let $t$ = the number of hours it takes Ladeen and Larry to complete their snow plowing route.

\[ \frac{7}{24} = \frac{1}{t} \]

\[ 7t = 24 \times 1 \]

\[ 7t = 24 \]

\[ t = \frac{24}{7} = 3.43 \]

Working together Ladeen and Larry can complete their snow plowing route in approximately 3.43 hours or about 3 hours 26 minutes.

4. \[ \text{Tagged polar bears in sample} \]

\[ \frac{\text{Total polar bears in sample}}{\text{Tagged polar bears in region}} = \frac{\text{Tagged polar bears in region}}{\text{Total polar bears in region}} \]

Let $p$ = the population of polar bears in the region.

\[ \frac{4}{10} = \frac{15}{p} \]

\[ 4p = 10 \times 15 \]

\[ 4p = 150 \]

\[ p = 37.5 \]

We estimate the polar bear population of the region to be 38.

5. \[ \text{percent} = \frac{\text{part}}{\text{whole}} \]

\[ \frac{18}{40} = .45 = 45\% \]

18 is 45% of 40.
11. Let \( x \) = the amount of almonds (in pounds) and let \( y \) = the amount of peanuts (in pounds).

\[
\begin{align*}
x + y &= 4 & \text{Weight equation} \\
7.75x + 3.25y &= 19 & \text{Money equation}
\end{align*}
\]

We chose to solve the system by substitution. Solving the first equation for \( y \) in terms of \( x \):

\[
y = 4 - x
\]

Substituting \( 4 - x \) for \( y \) into \( 7.75x + 3.25y = 19 \):

\[
7.75x + 3.25(4 - x) = 19
\]

Solving for \( x \):

\[
7.75x + 13 - 3.25x = 19
\]

\[
4.5x = 6 \quad \Rightarrow \quad x = \frac{6}{4.5} = \frac{4}{3}
\]

Substituting \( \frac{4}{3} \) for \( x \) into \( y = 4 - x \):

\[
y = 4 - \frac{4}{3} = \frac{8}{3}
\]

There are \( \frac{4}{3} \) pounds of almonds and \( \frac{2}{3} \) pounds of peanuts in the mix.

12. Let \( x \) = the amount of money (in dollars) invested in AAA bonds and let \( y \) = the amount of money (in dollars) invested in a C.D.

\[
\begin{align*}
x + y &= 5000 & \text{Investment equation} \\
.075x + .05y &= .065(5000) & \text{Interest equation}
\end{align*}
\]

We chose to solve the system by elimination.

\[
\begin{align*}
-.075(x + y) &= -.075(5000) \\
.075x + .05y &= 325
\end{align*}
\]

\[
-.025y = -50 \quad \Rightarrow \quad y = 2000
\]

\[
x + 2000 = 5000 \quad \text{Substitute 2000 for } y \quad \text{into } x + y = 5000
\]

\[
x = 3000
\]

Fred should invest $3000 in the AAA Bonds and $2000 in the C.D.

13. Let \( x \) = the amount of water (in L) added to the juice and let \( y \) = the amount of the mixture (in L).

\[
\begin{align*}
2 + x &= y & \text{Mixture equation} \\
.15(2) + .08y &= .08y & \text{Pure juice equation}
\end{align*}
\]

Solve the second equation for \( y \):

\[
.3 = .08y \quad \Rightarrow \quad y = \frac{3.75}{0.08} = 46.875 \quad \text{L}
\]

The size of the new product is 3.75 L. The old product was 2 L. Therefore 3.75 - 2 or 1.75 L of water was added.