Problem Solving - CRN 42087/42116/42132 - MATH 120/220/320 - 02

Department of Mathematical Sciences at CCSU

READ THIS SYLLABUS CAREFULLY. YOU ARE RESPONSIBLE FOR KNOWING THIS INFORMATION!

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Office Hours: TRF 10:30 a.m. - 12:30 p.m. or by appointment.

Prerequisites: These are distributed as follows:
For MATH 120: Math 115 or 119 with a grade of C- or higher or Placement Exam.
For MATH 220: Math 120 and 152 with a grade of C- or higher.
For MATH 320: Math 220 and 228 with a grade of C- or higher.

Students for Whom the Course is Intended: Secondary Mathematics (B.S.) majors. This is one of three one-credit seminars (MATH 120, MATH 220, and MATH 320), which are required for the B.S. Secondary Mathematics majors. It is also recommended for B.S. Elementary Mathematics majors to be taken as an elective.

Course Description: Polya’s four-step approach to problem solving applied to non-routine problems in algebra, geometry, and trigonometry. Strong emphasis placed on clarity, comprehensiveness, and correct use of mathematical terminology in student solutions. One two-hour lab per week.

Class Meetings: 10:50 a.m. - 12:30 p.m.; M; Henry Barnard Hall 329.

Textbook: There is no textbook for this course. Your professor will provide you with handouts containing problems to be worked on. In addition, a list of resources for additional problems is attached to this syllabus.

Attendance: Since this is a seminar, which meets only once a week and interaction among students is an essential part of the course, attendance at every class is expected. Absences will adversely affect the class participation portion of the final grade.

Work Load: The norm for university courses is that a minimum of two hours of homework per week is required for every class hour. Consequently, for this class you are expected to spend at least 3 hours per week outside of class. However, since the final grade is based only on the quality of work, some students may need many hours of study.

Expectations: All three seminars, MATH 120, MATH 220, and MATH 320 meet at the same time. Students in MATH 220 and MATH 320 will have had the experience of taking this seminar at least once previously. In addition, in most cases they will have completed more of the standard courses in the major. For instance, all students in MATH 220 will have at least one semester in calculus. Students in MATH 320 will have completed courses in discrete mathematics and linear algebra. The mixing of the three groups is deliberately designed to enable students in MATH 120 to learn from more experienced problem solvers.

Because of the heterogeneous nature of the class, expectations are set on an individual basis. You will be given a wide variety of problems to choose from. Work on those that you find challenging, but not impossible. The problems will be weighted according to the level of
difficulty. The easiest problems will be worth 1 point each, more difficult problems 2 points each and so on.

**Portfolios:** On four occasions during the semester, you will submit a portfolio containing detailed solutions to some problems you have worked over the past few weeks. The problems submitted must carry a total weight of at least 6 points for Math 120 students, 7 points for Math 220, and 8 points for Math 320. Along with each solution you will reflect on the process you went through to solve the problem, using Polya’s four steps as a framework. We are particularly interested in problems that prove challenging to you, those for which your initial approach was unfruitful, and situations that lead to significant insights and discoveries. The problems submitted in the portfolio may include some you have worked on by yourself and some you have discussed with classmates. Please acknowledge instances where other class members or the instructor have contributed to the solution or where you have used hints provided by the author of the textbook or other sources. Portfolios will be assessed for the level of difficulty of the problems, the accuracy and elegance of the solutions, and your analysis of the problem solving process. Note: The nature of the problems given DO NOT lend themselves to calculator use. All portfolio solutions should be typed. You must use the Equation Editor or LaTeX and Geometer’s Sketchpad or Geogebra for a professional looking document. A sample-grading sheet is attached to this syllabus.

Each portfolio must be stapled with a cover sheet that contains the following information:

- **Your name:**
- **MATH 220**
- **Problems submitted:**
  - #4  2 points
  - #10  2 points
  - #27  3 points
- **total 7 points**

**ALSO:**
- Begin each problem at the top of a new page. Place the problem solutions in numerical order. Submit your grade sheet with each portfolio.

**Final Presentation:** In place of a final examination, you will choose one problem to present to the class on April 25, May 2, or May 9, 2016. The problem you choose may come from any of the four problem sets that you will be given this semester. You will give a ten-minute oral presentation explaining your solution. A rubric for the presentation evaluations is provided on the last page of this syllabus.

**Class participation:** Your class participation will be graded. An important part of this grade will be oral presentations of problems that carry at least five points and are **not included** in your portfolios. For these presentations you must choose a question from one of the first three problem sets. You are advised to make these presentations early and throughout the course, and must complete these presentations no later than April 18, 2016.

**Reading report:** The report is due on May 2, 2016. Visit the Elihu Burritt Library and browse the resources held on reserve for this class. For writing this report you are **NOT** allowed to use any other resources! Based on your readings you write a report on topics you found of interest. The aim is to encourage you to pursue problem solving topics of interest to you, improve your writing and critiquing of mathematical prose, and enable you to use the mathematics resources in the libraries.
The report should be about 5-10 pages long and should not simply be a revision of another report of the same genre and length. Every direct quote must, of course, be credited, as must paraphrasing. You might find a large piece of your report ending up as a paraphrase of one source. It is acceptable to say this at the start of such a segment rather than citing every sentence or paragraph. For example, you might say, The material on pages 3 and 4 is basically a digestion of Brown [1, 45-50]. In all cases, do not take the risks of plagiarism. The following comment should guide your critique of your own writing.

Audience. The students of this class are the intended audience. Three to six minutes of dedicated reading per page should yield a decent understanding. I encourage each of you to pick a topic that will stretch your mathematical abilities. At the same time, I think topics relevant to high school teaching are quite appropriate, pursuing connections with other mathematics, applications and/or relevant pedagogical issues.

Format. Use a word processor to facilitate rewriting. Use 12 point font. Typeset math symbols are nice, but hand lettered symbols are fine. Draw hand made symbols and figures with black ink. Leave enough space for symbols and figures. Formulas should be displayed on their own lines and numbered if you will refer to them beyond the immediately following or preceding sentences. The first page should start with the title and your name. Number each page. It is standard to give the authors last name followed by the number of that item in your bibliography, enclosed in brackets. For example, Brown [1, 45-50]. Use clear, concise, correct English. Develop concepts rather than grind through unenlightening details.

**Evaluation:** Your grade for the course will be determined on the following basis:

- Class participation: 15%
- Portfolios (four, 15% each): 60%
- Reading report: 10%
- Final presentation: 15%
- Total: 100%

Letter Grades will still be assigned according to the following standard 100-point scale:

- A: 92.5 and above
- A-: 89.5-92.4
- B+: 86.5-89.4
- B: 82.5-86.4
- B-: 79.5-82.4
- C+: 76.5 - 79.4
- C: 72.5-76.4
- C-: 69.5-72.4
- D+: 66.5-69.4
- D: 62.5-66.4
- D-: 59.5 - 62.4
- F: 59.4 and below

**University Policies:** 1. Please contact me privately to discuss your specific needs if you believe you need course accommodations based on the impact of a disability, medical condition, or if you have emergency medical information to share. I will need a copy of the accommodation letter from Student Disability Services at least two days before the accommodation is needed in order to arrange your class accommodations. Contact Student Disability Services, room 101, Willard Hall if you are not already registered with them. Student Disability Services maintains the confidential documentation of your disability and assists you in coordinating reasonable accommodations with your faculty. Note my contact information given on the first page.

2. In the event of a weather emergency which requires curtailment or cancellation of classes, listen to WTIC (1080 AM) or call (860) 832-3333 for the “general message.” You can also check on the main CCSU website under “Cancellations and Delays”.
3. The last day to withdraw from a course is Monday, Apr 18. Approvals for withdrawal are not required; however, it is strongly recommended that students consult with their academic advisors prior to deciding to withdraw. Cessation of attendance, notice to the instructor, or telephone calls to the Enrollment Center are not considered official notice of a student’s intention to drop the course. After Apr 18 withdrawals are allowed only under extenuating circumstances and require approval of the course instructor, department chair and dean of the School of Engineering, Science & Technology. Poor academic performance is not considered an extenuating circumstance.

4. You are responsible for understanding and abiding by the University’s policy on academic integrity. Please be careful! The internet is a useful place for information, but a dangerous place for risk of plagiarism. Do not tempt it. Not only will you not learn and thus fail the course anyway, you will also get yourself in trouble. Information on the University’s policy may be found at http://www.ccsu.edu/AcademicIntegrity/. This policy is rigorously enforced by your instructor and by the Department of Mathematical Sciences.

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<th>Week</th>
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<th>Monday</th>
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<tr>
<td>1</td>
<td>Jan 25</td>
<td>class</td>
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<td>2</td>
<td>Feb 1</td>
<td>LaTeX &amp; GeoGebra</td>
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<td>Feb 8</td>
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<td>8</td>
<td>Mar 7</td>
<td>Portfolio #2 Due</td>
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<td>9</td>
<td>Mar 14</td>
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<td>11</td>
<td>Mar 28</td>
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<td>12</td>
<td>Apr 4</td>
<td>Portfolio #3 Due</td>
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<td>13</td>
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<td>14</td>
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<td>16</td>
<td>Apr 25</td>
<td>Final Presentation, Portfolio #4 Due</td>
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<td>May 2</td>
<td>Final Presentation, Reading Report Due</td>
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<td>17</td>
<td>May 9</td>
<td>Final Presentation (11a.m.-1 p.m.)</td>
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Resources

In order to become a skilful problem solver you MUST READ A LOT! Many of the books below are on reserve in the Elihu Burritt Library and you should take advantage of the reading room. Some links are quite useful to introduce you to reading and writing mathematics as well as to the culture of problem solving.

BOOKS


Polya, George. *How to Solve It*.

MEDIA

Paul Zeitz. *The Art and Craft of Problem Solving*. 4 DVDs

WEB LINKS

GeoGebra [http://www.geogebra.org](http://www.geogebra.org)
LaTeX [https://latex-project.org/intro.html](https://latex-project.org/intro.html)

“Students think that all mathematics is known and, like Latin grammar, must be rehearsed until it is learned. More importantly, they have no idea that “understanding” mathematics means asking questions until things make sense; instead, it means passively reproducing what they have been shown. [...] it is disturbing that college freshmen do not routinely think to draw diagrams to help them understand problem statements, test hypotheses with special cases, etc. Even more disturbing is the fact that my students rarely realize that they can think, that they can watch themselves thinking, and that they can improve their problem solving performance by reflecting on their successes and failures. What seems perfectly natural in the context of playing tennis, or any other sport, seems completely alien in the context of training one’s mind!”


“A Handy List of Questions:

- Is there a formula?
- What is the formula?
- What purpose does the formula serve?
- What is the number of objects or cases satisfying this condition?
- What is the maximum?
- What is the minimum?
• What is the range of the answer?
• Is there a pattern here?
• What is the pattern in this case?
• Is there a counterexample?
• Can it be extended?
• Does it exist?
• Is there a solution?
• Can we find the solution?
• How can we condense the information?
• Can we make a table?
• Can we prove it?
• When is it false? When true?
• Is it constant?
• What is constant, what is variable?
• Does it depend on something we can specify?
• Is there a limiting case?
• What is the domain?
• Where does the proof break down in an analogous situation?
• Is there a uniting theme?
• Is it relevant?
• Are we imposing any restrictions without intending to do so?
• When is it relevant?
• What does it remind you of?
• How can one salvage what appears to be a breakdown?
• How can you view it geometrically?
• How can you view it algebraically?
• How can you view it analytically?
• What do they have in common?
• What do I need in order to prove this?
• What are key features of the situation?
• What are the key constraints currently being imposed on the situation?
• Does viewing actual data suggest anything interesting?
• How does this relate to other things?"


“Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jump into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, Does this make sense?"

[Common Core State Standards for Mathematics, 2010, page 6.]
“MP: That brings up my next question. How did you become interested in heuristics and the art of problem solving? Did anyone or any event influence you on this?

Polya: Well, I think I wrote it somewhere. In one of my books it is mentioned. I came very late to mathematics. I had an interest in biology, literature and philosophy. And as I came to mathematics and learned something of it, I thought: Well, it is so, I see, the proof seems to be conclusive, but how can people find such results? My difficulty in understanding mathematics: How was it discovered? And then I was deeply influenced by some books. I wish to mention just two. One was the book of Ernst Mach on the history of mechanics. For me personally this was the most beautiful book I read. I read it at the right time because I knew a little physics, but just a little. I was just right for it. His main theme is: You cannot understand a theory unless you know how it was discovered. His best book and best-known book is on mechanics, but he wrote also other books, on the theory of heat and still others. But that was the main idea: In order to understand a theory really, you must know how it was discovered. So he came to heuristics. In fact, in some of his other books there are a few direct remarks on problem solving. Then I thought about it and I came across the Regulae of Descartes, which is really a book on problem solving. That is not mentioned in any history of philosophy, because those historians who wrote about him didn’t know about problem solving. My interest in literature contributed a little. When I was interested in literature, I was most interested in books of Hippolyte Tame and he wrote about literature in a quasi-scientific way. How in such a vague subject you can bring in something that approximates science, that deeply impressed me. It also contributed to my interest in heuristics. It is essentially a vague question, and that you can introduce something which has something to do with science, that I think I learned from Tame. I was also impressed by his style.”

[George Polya was interviewed by G.L.Alexanderson at Polya’s 90th birthday party at Stanford, May 13, 1977. You can read more about this interview in Mathematicical People: Profiles and Interviews. Burkhauser Boston, Inc., 1985. The pictures are from this source too.]

Polya’s problem solving steps:

First. You have to understand the problem. Do you understand all the words used in stating the problem? What are you asked to find or show? Can you restate the problem in your own words?

UNDERSTANDING THE PROBLEM

What is the unknown? What are the data? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory? Draw a figure. Introduce suitable notation. Separate the various parts of the condition. Can you write them down?

Second. Find the connection between the data and the unknown. Guess and check, make an orderly list, eliminate possibilities, use symmetry, consider special cases, use direct reasoning, solve an equation, look for a pattern, draw a picture, solve a simpler problem, use a model, use a formula, be ingenious.

DEVISING A PLAN

Have you seen it before? Or have you seen the same problem in a slightly different form? Do you know a related problem? Do you know a theorem that could be useful? Look at the
unknown! And try to think of a familiar problem having the same or a similar unknown. Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible? Could you restate the problem? Could you restate it still differently? Go back to definitions. If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other? Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

Third. Carry out your plan. Persist with the plan that you have chosen. If it continues not to work discard it and choose another.

**CARRYING OUT THE PLAN**

Carrying out your plan of the solution, check each step. Can you see clearly that the step is correct? Can you prove that it is correct?

Fourth. Examine the solution obtained. Doing this will enable you to predict what strategy to use to solve future problems.

**LOOKING BACK.**

Can you check the result? Can you check the argument? Can you derive the result differently? Can you see it at a glance? Can you use the result, or the method, for some other problem?

[Problem solving heuristics from George Polya, Princeton University Press, 1973]

![Figure 1. G. Polya (1887-1985), Professor of Mathematics at Stanford University](image-url)