Student-directed exploratory learning, multiple perspectives, mathematical communication, and the development of mathematical thinking clearly are encouraged by the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989). The Standards are based on an enlightened philosophy of learning and are supported by a rapidly growing body of research (Kamii 1994; Wood and Sellers 1997; Carroll 1997). However, meeting the Standards is a difficult challenge for many teachers of mathematics.

Prescribed textbook-based curricula; expectations of students, parents, and administrators; inadequate training in facilitating learning; and absence of support materials are some obstacles that must be overcome. However, the toughest obstacle may reside within the teachers themselves. This article describes one teacher’s effort to change the classroom environment. It reveals a teacher, strongly motivated by dissatisfaction with existing practices, experimenting with a new approach to teaching. It shows that change can be made. The story is told through the teacher’s perception of classroom events.

### THE TEACHER’S STORY

Three factors drew me into this situation: First, I was not satisfied with the traditional “explain-practice” classroom routine even when it was enhanced with discussion and group work. Second, the students’ lack of mathematical reasoning bothered me. Third, the Lesson Graph computer program seemed to offer intriguing possibilities.

I hoped to use Lesson Graph to help students improve their thinking strategies and to help them learn to graph linear equations. When I explored the program, it had given me a new perspective of how coefficients of a linear equation affect the location of its line on the graph. But since the program offers no instruction on graphing equations, I wondered whether students could gain similar insights.

In this lesson, students view the screen shown in figure 1. They enter values for $a$, $b$, and $c$ in the general equation $aX + bY = c$ then click PLOT, and a colored line appears on the graph. The equation appears in the same color next to the line. Students can erase either the most recent line graphed or all the lines.

After a successful pilot test with advanced seventh graders, the real test class was a typical ninth-grade, first-year-algebra class. The students

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were accustomed to learning through traditional textbook methods. They knew the general form of linear equations, $aX + bY = c$, and could graph linear equations in slope-intercept form by first making a table.

The class had access to the computer lab only during the first thirty minutes of the period. Before using the lab, we discussed conjectures by studying examples. I introduced the computer program and challenged the students to determine the effects of $a$, $b$, and $c$ on the graph of $aX + bY = c$.

**Day 1—lab**

I reminded students that their task was to determine where the line would appear on the graph, depending on the coefficients in $aX + bY = c$. I instructed them to experiment and make conjectures on the basis of their experiments and observations. The students were to work in pairs. For this lab period, I did not answer students’ questions directly or give hints. I instead asked them to tell me what they had tried and what they might try next.

I strongly emphasized the value of conjectures, even though we might reject some later. I also stressed that we would build on one another’s ideas in the way that mathematicians have worked for centuries.

As students began their explorations, the room was much louder and far more chaotic than usual. This classroom experience was the students’ first one in which they were asked to discover mathematics. I wondered whether they would learn. Adding to my growing anxiety was concern that the class would fall behind the other first-year-algebra classes. Yet my experience with similar simulations in a computer-programming class indicated that, if successful, this approach could be superior to the traditional approach.

The following dialogue captures the initial climate.

**Mary:** [shouting from across the lab] Mr. Thomas, what are we supposed to do?

**Me:** [noticing the empty graph on her screen] I want you to discover a way to graph an equation without having to make a table of values. You need to find out what effect the coefficients have on the graph of that equation. (This statement summarized previous group instructions.)

**Mary:** [anxiously, as I began turning away] Mr. Thomas, how are we supposed to do that?

**Me:** You need to enter a few equations and observe their graphs. When you believe that you see a relationship, write down your conjecture.

**Mary:** But, Mr. Thomas, what do you want us to do?

**Me:** Type in $2X + 3Y = 6$, and click on PLOT. (Again, this statement repeated a previous suggestion.)

**Sarah:** [impatiently] Come on, let’s just do something. He’s not going to tell us.

In less than five minutes, the students decided that the value of $a$ affected the slant of the line. Most of the eleven pairs of students accepted the task and began making good conjectures. One pair seemed to give up, and two more became distracted by off-task conversations. Another pair, who seemed lost as to where to begin, stared at their screen for a few minutes. Then they began in earnest to create superficial work to have something for which I might give credit.

**Day 1—classroom**

Students’ conjectures were listed on the chalkboard for discussion. As usual, two or three students volunteered to share, whereas others had to be coaxed. Some of the conjectures were as follows:

1. When $a = 2$, $b = 0$, and $c = 6$, the line is vertical.
2. If you switch the numbers for $a$ and $b$, the line goes through the $x$- and $y$-axes at opposite places.
3a. The coefficient of $x$ controls the angle of the line.
3b. As $x$ goes up, the line gets steeper.
4a. Putting in zero for the $x$-coefficient makes the line horizontal.
4b. Making the $x$-coefficient zero makes the line horizontal, and making the $y$-coefficient zero makes it vertical.
5. When you change the $x$-coefficient, all lines go through the same place on the $y$-axis.

In viewing these conjectures, we focused on the student’s strategy rather than on the conjecture’s accuracy. Most discoveries appeared to result from random trial-and-error approaches, several conjectures held for more general cases than the one stated, and some conjectures were more helpful than others. I pushed the discussion toward conjecture power, symmetry, the usefulness of zero, isolating variables, and learning from others.

Comparing conjecture (3a) with conjecture (3b) and conjecture (4a) with conjecture (4b) led to a discussion of which conjectures should be kept, which could be discarded, and the value of striving for more powerful conjectures. In discussing conjectures (4a) and (4b), students noticed that an observation about the $x$-coefficient suggested a similar relationship for the $y$-coefficient. This realization encouraged improvements in other conjectures as students pointed to conjectures (3b) and (5). Conjecture (5) was changed to “Changing only the coefficient of $x$ causes all lines to cross the $y$-axis in the same place. And changing only the coefficient of $y$ causes all lines to cross the $x$-axis in the same place.” This conjecture was a major step forward.
and was suggested as a guide for lab explorations the following day. The class also wrestled with questions of why zero makes a vertical or horizontal line and the investigative usefulness of changing one coefficient at a time.

Day 2—lab
I gave the students a sheet of paper on which to record their conjectures. The following information appeared on the top of this sheet:

Remember: Your goal is to find an easier way than making a table of values to graph equations of the form \(aX + bY = c\). (For example, \(2X + 3Y = 6\).)

Conjecture: Changing only the coefficient of \(x\) causes all lines to cross the \(y\)-axis in the same place.

Conjecture: Changing only the coefficient of \(y\) causes all lines to cross the \(x\)-axis in the same place.

The second day in the lab went more smoothly than the first. Students were more focused, quieter, and on task. They used more systematic approaches and made more powerful conjectures. The anxious and demanding tone of students’ questions was gone. A couple of students seemed mildly annoyed at the distraction when I checked their progress.

Day 2—classroom
When invited to share their conjectures, about a third of the students responded. Each seemed eager to share a discovery before a peer revealed the same observation. There were numerous “me, too’s.” Some of their conjectures follow:

1. When \(b\) and \(c\) stay the same, the line crosses at the same place on the \(y\)-axis. (Several students stated that when \(a\) and \(c\) remain the same, the same result happens on the \(x\)-axis.)
2. When \(b\) and \(c\) stay the same, [the line] goes through \((0, 4)\).
3. As \(c\) increases, the line moves farther to the right.
4. \(2X + 3Y = 6, 4X + 6Y = 10\), and \(6X + 9Y = 15\) are all parallel.
5. The place where the line crosses the \(y\)-axis is at \(c/d\) divided by \(b\).
6. Changing the value of \(c\) makes the line cross the \(y\)-axis at different places but does not change the slant.

Since conjectures (5) and (6) indicated that some students had not internalized the previous day’s discussion on symmetry, I asked whether conjecture (5) left any nagging questions. A student who rarely participated in discussions volunteered that the \(x\)-intercept was located at \(c/a\). This statement led to a discussion of the other conjectures with respect to power, symmetry, and strategies of discovery. For homework, students were asked to predict where the graph of \(2X + 5Y = 10\) would appear. I also stated that all the lines seen thus far rose to the left and passed through the first quadrant or were vertical or horizontal, and I asked students to consider other lines that might appear on a graph.

Day 3—lab
Students used the computer to check their homework. To encourage them to expand their concept of intercept, I then challenged them to make a line that crossed the \(y\)-axis at \((0, 2)\) but that crossed the \(x\)-axis at \((-5, 0)\). Through some cooperative efforts, all pairs explored the effects of negative coefficients.

Day 3—classroom
We discussed scientific thinking at length, and I asked students how they would search for answers to the following questions:

- How can we tell from an equation whether the graph of the line will rise to the right or rise to the left?
- In addition to the coefficients, what else that appears in an equation might we change?

We started with “1. \(2X + 3Y = 6\)” written on the chalkboard. The students then suggested comparing the graph of this equation to those of the following:

2. \(-2X + 3Y = 6\)
3. \(2X - 3Y = 6\)
4. \(-2X - 3Y = 6\)
5. \(-2X - 3Y = -6\)
6. \(2X + 3Y = -6\)

Only after all six equations were listed on the chalkboard did I number any of them.

Volunteers quickly graphed each of these equations on the chalkboard, and the class made two additional conjectures: when a line rises to the right, \(a\) and \(b\) have opposite signs; and when a line rises to the left, \(a\) and \(b\) have the same signs. Students observed that equation (5) produced the same line as equation (1), and equation (6) produced the same line as equation (4). This observation led one student to suggest that some of these equations must actually be the same as others even though they did not appear to be so. It interested me that students did not apply the distributive property on their own and notice that fact, even though most had applied it in solving equations. This connection would be essential in transferring their knowledge of graphing equations in general form to graphing slope-intercept form.
OBSERVATIONS
Class discussions during this unit were unique. All twenty-two students were attentive and tried to discover relationships. Two students who had previously tried to sleep in class contributed throughout the entire period, as did students who had seemed lost and frustrated. Instead of two or three students trying to answer all the questions, five or six were now competing for this opportunity. I could call on any student and receive an answer that helped move the class toward a solution.

RESULTS
This activity did not cure all ills. During the remaining four weeks of the semester, students still made as many careless errors as before, but they used far better learning strategies. They were able to get further on their own with test and homework problems. They were less dependent on my telling them how to solve problems and more willing to search for answers on their own. Two students who had frequently tried to distract the class never did so again. My concern with falling behind other algebra classes was unnecessary. These students quickly grasped material in the units that remained, and their scores improved slightly in comparison with the scores of my other class.

My greatest pleasure in teaching this unit came from the success of one student. Before this lesson, the student had worked very hard but had not grasped the concepts. Giving her alternative explanations and tutoring outside of class had not helped. She had received a D for the first quarter, a fact that disturbed me in view of her effort and frustration. During this unit, her conjectures were the most insightful in the class. She was the first to find the intercepts and the first to experiment with zero. Even though it was late in the grading period, she raised her grade to a B– in second quarter and carried a B through second semester. She felt good about herself again.

In using this software, I was surprised by the new type of learning environment—the questions, the problems that students encountered, and their successes. I am aware of many mistakes that I made, mostly in not taking advantage of an opportunity to push a student’s insight or learning strategy another step forward. I believe that five or six of these types of experiences over the course of a year would greatly enhance students’ understanding of mathematics and my ability to teach it.

CONCLUSION
Helping students learn how to learn mathematics is central to the NCTM’s Standards. This article describes a learning environment that is designed to implement the Standards. To benefit from this environment, both the students and the teacher had to change their expectations and practices. The students initially resisted taking responsibility, but once this obstacle was overcome, most students viewed mathematics and learning from a new and more valuable perspective. The teacher’s major challenge was implementing higher-level goals.

In most lessons, goals pertain to task, discipline, and learning how to learn. In using Lesson Graph, task-level goals include finding the x- and y-intercepts and plotting lines. Discipline-level goals include recognizing and using symmetry. The most important goals—and those most challenging to teachers and students—involves learning how to learn mathematics. The classroom experience with this graphing software was designed to give highest priority to building learning strategies.

The strategies of controlling variables, making and testing conjectures, and building on the knowledge of others were foreign to these students. With this lesson, the teacher was challenged to anticipate, recognize, and capitalize on all the valuable learning opportunities that arose. Developing insightful questions while creating an environment for encouraging independent learning, multiple perspectives, mathematical communication, and the development of mathematical thinking is the mountain that teachers must climb.

REFERENCES